A New Algorithm for Measuring and Optimizing the Manipulability Index

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Abstract The estimation of the performance characteristics of robot manipulators is crucial in robot application and design. Furthermore, studying the manipulability index for every point within the workspace of any serial manipulator is considered an important problem. Such studies are required for designing trajectories to avoid singular configurations. In this paper, a new method for measuring the manipulability index is proposed, and then some simulations are performed on different industrial manipulators such as the Puma 560 manipulator, a six DOF manipulator and the Mitsubishi Movemaster manipulator.

Keywords Manipulability index · Dexterity · Robot manipulator

1 Introduction

Studying the performance characteristics of the robot such as dexterity, manipulability, and accuracy is very important to the design and analysis of a robot manipulator. The manipulability is the ability to move in arbitrary directions while the accuracy is a measure of how close the manipulator can return to a previously taught point. The workspace of a manipulator is a total volume swiped out by the end effector when it executes all possible motions. The workspace is subdivided into the reachable workspace and the dexterous workspace. The reachable workspace is all point reachable by the end-effector. But the dexterous workspace consists of all points

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that the end-effector can reach with an arbitrary orientation of the end-effector. Therefore, the dexterous workspace is a subset of the reachable workspace. The dexterity index is a measure of a manipulator to achieve different orientations for each point within the workspace.

In this paper, we present a new method for measuring the manipulability index, and some simulations are implemented on different manipulators such as the Puma 560 manipulator, a six DOF manipulator and the Mitsubishi Movemaster manipulator. In addition, we describe how the manipulability measure is crucial in performing intelligent behavior tasks. The manipulability index is considered as a quantitative and performance measure of the ability for realizing some tasks. This measure should be taken into consideration in the design phase of a serial robot and also in the design of control algorithms. Furthermore, we use the proposed method for measuring the manipulability index in serial manipulators to generalize the standard definition of the manipulability index in the case of mobile manipulators.

2 Prior Work

Charles Klein and Bruce Blaho [1] proposed some measures for the dexterity of manipulators, then they compared several measures for the problems of finding an optimal configuration for a given end-effector position, finding an optimal work-point, and designing the optimal link lengths of an arm. They considered four measures for dexterity: determinant, condition number, minimum singular value of the Jacobian and joint range availability. Salisbury and Craig [2] illustrated hand designs with particular mobility properties. In addition, they gave a definition of accuracy points within manipulator workspace. They used another performance index which is the condition number of the Jacobian. Yoshikawa [3] gave one of the first mathematical measures for the manipulability of any serial robot by discussing the manipulating ability of robotic mechanisms in positioning and orienting end-effectors. He introduced the term manipulability, which involves the Jacobian and its transpose; then the evaluation of the determinant of the Jacobian can be used to determine the manipulability measure. Gosselin [4] presented two dexterity indices for planar manipulations, the first one is based on a redundant formulation of the velocity equations and the second one is based on the minimum number of parameters. Then the corresponding indices were derived for spatial manipulators. These indices are based on the condition number of the Jacobian matrix of the manipulators. He considered the dexterity index, manipulability, condition number and minimum singular value, then he applied these indexes to a SCARA type robot. Kees van den Doel and Dinesh K. Pai [5] introduced a performance measure of robot manipulators in a unified framework based on differential geometry. The measures are applied to the analysis of two- and three-link planar arm. In [6], the authors demonstrated that manipulability of a mechanism is independent of task space coordinates. Furthermore, they provided a proof of the independency of the manipulability index on the first DOF. In [7], the author examined two geometric tools for measuring the dexterousness of robot manipulators, manipulability ellipsoids and manipulability polytopes. He illustrated that the manipulability ellipsoid does not transform the exact joint velocity constraints into task space and so may fail to give exact dexterousness measure and optimal direction of motion in task

space. Furthermore, he proposed a practical polytope method which can be applied to general 6-dimensional task space.

In [8], Sobh and Toundykov presented a prototyping software tool which runs under the mathematica environment, automatically computes possible optimal parameters of robot arms by applying numerical optimization techniques to the manipulability function, combined with distances to the targets and restrictions on the dimensions of the robot.

Nearly all of the above techniques start by getting the forward kinematics then the Jacobian equation which relates the velocity of the end-effector and the joint velocities.

3 Manipulability Measure

3.1 Jacobian Matrix

The Jacobian matrix provides a transformation from the velocity of the end-effector in cartesian space to the actuated joint velocities as shown in Eq. 1

$$\dot{x} = J\dot{q} \tag{1}$$

Where $\dot{q}$ is an m-dimensional vector that represents a set of actuated joint rates, $\dot{x}$ is an n-dimensional output velocity vector of the end-effector, and J is the $m \times n$ Jacobian matrix. It is possible that $m \neq n$. As an example, a redundant manipulator can have more than six actuated joints, while the end-effector will at most have six degrees of freedom, so that m > n.

In the singular position, the Jacobian matrix J looses rank. This means that the end-effector looses one or more degrees of twist freedom (i.e., instantaneously, the end-effector cannot move in these directions). The mathematical discussion of singularities relies on the rank of the Jacobian matrix J, which, for a serial manipulator with n joints, is a $6 \times n$ matrix. For a square Jacobian, $\det(J) = 0$ is a necessary and sufficient condition for a singularity to appear.

3.2 Singular Value Decomposition Method

The Singular Value Decomposition method (SVD) works for all possible kinematic structures (i.e. with every Jacobian matrix J with arbitrary dimensions $m \times n$). The SVD decomposition of any matrix J is on the form:

$$J_{mxn} = U_{mxm} \Sigma_{mxn} V_{nxn}^t \tag{2}$$

With $\Sigma = \begin{pmatrix} \sigma_1 & 0 & 0 & \ldots & 0 & 0 \ldots 0 \\ 0 & \sigma_2 & 0 & \ldots & 0 & 0 \ldots 0 \\ 0 & 0 & \ddots & 0 & \ldots 0 \ldots 0 \\ 0 & 0 & \ldots & \sigma_{m-1} & 0 & 0 \ldots 0 \\ 0 & 0 & \ldots & 0 & \sigma_m & 0 \ldots 0 \end{pmatrix}$

Such that U and V are orthogonal matrices. Thus,

$$U^t U = I_{mxm} \tag{3}$$

$$V^t V = I_{nxn} \tag{4}$$

Where I is the identity matrix and the singular values are in Descending orders $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_m$. The matrix has a zero determinant and is therefore singular (it has no inverse). The matrix has two identical rows (and two identical columns). In other words, the rows are not independent. If one row is a multiple of another, then they are not independent, and the determinant is zero. (Equivalently: If one column is a multiple of another, then they are not independent, and the determinant is zero.) The rank of a matrix is the maximum number of independent rows (or, the maximum number of independent columns). A square matrix $A_{n \times n}$ is non-singular only if its rank is equal to n. Mathematically, matrix J having a full rank means that the rank of J = m. In this case, $\sigma_m \neq 0$. When $\sigma_m \approx 0$, the matrix J does not have a full rank, which means that the matrix J looses one or more degrees of freedom. This case happens physically, when the serial robot has two joint axes coinciding on each other.

3.3 Manipulability Measures

Yoshikawa [3] defined the manipulability measure μ as the square root of the determinant of the product of the manipulator Jacobian by its transpose

$$\mu = [\det(J \cdot J^t)]^{1/2} \tag{5}$$

If the Jacobian matrix J is a square matrix, the manipulability μ is equal to the absolute value of the determinant of the Jacobian. Using the singular value decomposition the manipulability can be written as follows:

$$\mu = \sigma_1 \sigma_2 \cdots \sigma_m \tag{6}$$

Another method for the manipulability measure is the reciprocal of the condition number [termed the conditioning index] was used in [9].

3.4 Optimizing the Manipulability Index of Serial Manipulators Using the SVD Method

Our current work addresses the manipulability index for every point within the workspace of some serial manipulators. The method provided promising results,

Table 1 DH parameters of the Puma 560 manipulator

i	α	a	θ	d	Initial limit	Final limit	Joint's type
1	90	0	*	0	−170	170	R
2	0	0.4318	*	0	−225	45	R
3	−90	0.0203	*	0.15005	−250	75	R
4	90	0	*	0.4318	−135	100	R
5	−90	0	*	0	−100	100	R
6	0	0	*	0	−180	180	R

Table 2 DH parameters of a six degrees of freedom serial manipulator

i	α	a	θ	d	Initial limit	Final limit	Joint's type
1	90	0	*	10	−170	170	R
2	0	10	*	0	−225	45	R
3	−90	0	*	0	−250	75	R
4	90	0	*	10	−135	100	R
5	−90	0	*	0	−100	100	R
6	0	0	*	0	−180	180	R

since it is considered one of the crucial tasks required for designing trajectories or avoiding singular configurations. We propose a new method for measuring the manipulability, then, we implemented simulations supporting our method on the Puma 560 manipulator, a six DOF manipulator and the Mitsubishi Movemaster manipulator. As mentioned in [9], the determinant of a Jacobian cannot be used for expressing the manipulability's index. It reaches zero when a manipulator reaches any singular configuration. Another method has been proposed, labeled the reciprocal of the Jacobian (as in [9]). In past research, there was an argument about whether the minimum value of the σ's in Eq. 2 or the multiplication of all σ's exactly represent the manipulability's index[4].

In this work, we propose a new concept for measuring this index, then justify this concept by visualizing the bands of this index, resulting from our experiments. Moreover, a new relationship between the minimum rank of the Jacobian matrix, and the order of one of these σ's (in Eq. 2) that can exactly express the manipulability's index.

3.4.1 The Puma 560 Manipulator: A Case Study

In case of the singular configuration of the Puma 560 manipulator at Q= $[0, 0, -\frac{\pi}{2}, 0, 0, 0]$, the following would be the J, U, Σ and V matrices as depicted in Eq. 2:

$$J = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Table 3 Manipulability's bands of the Mitsubishi Movemaster manipulator in 2-D workspace

i	α	a	θ	d	Initial limit	Final limit	Joint's type
1	90	0	*	300	−150	150	R
2	0	250	*	0	100	130	R
3	0	160	*	0	−110	0	R
4	−90	0	*	0	−90	90	R
5	0	0	*	72	0	0	R

$$U = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -0.0034 \\ 0 & -0.9988 & 0 & 0 & -0.0002 & -0.0499 \\ -0.9982 & 0 & 0 & 0.06 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0.06 & 0 & 0 & 0.9982 & 0 & 0 \\ 0 & -0.0499 & 0 & 0 & 0.0034 & 0.9987 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 22.401 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20.025 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.4142 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0935 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ -0.8939 & 0 & 0 & 0.1852 & 0.4074 & -0.027 \\ -0.4483 & 0 & 0 & -0.3638 & -0.8147 & 0.0539 \\ 0 & 0 & -0.7071 & 0 & -0.0467 & -0.7056 \\ -0.0027 & 0 & 0 & -0.9129 & 0.4074 & -0.027 \\ 0 & 0 & -0.7071 & 0 & 0.0467 & 0.7056 \end{bmatrix}$$

It is obvious that in the singular matrix σ; σ_5 and σ_6 assume the value zero with small tolerance. This is due to the fact that there are two singular cases in its configuration; the fourth and sixth joints are on same axis and it is in a singular arm configuration, thus σ_5 is zero.

3.5 Proposed Manipulability Measure Algorithm

To justify the proposed method, the following algorithm is proposed:

**Algorithm 1** Calculate the manipulability index

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1: Find the joint(s) that may lead to a singular configuration assuming that the number of these joints = n.
2: **for** $i = 1$ to n **do**
3:     Change the value of the ith joint from its initial to its final value using simulation software - Matlab robotic toolbox [10] is used in our case.
4:     Calculate the Jacobian (J) and singular (Σ) matrix.
5:   Plot every normalized σ and also the rank of the Jacobian matrix.

$$Normalized \sigma_i = \frac{\sigma_i}{Max\{\sigma_{i1}, \sigma_{i2}, \sigma_{i3}, ..., \sigma_{in}\}} \tag{7}$$

    Where: i is the order of the σ in the singular matrix and n is the number of steps during the simulation.
6:     Check the rank of the Jacobian matrix.
7: **end for**

Fig. 1 Phases for the Puma 560 manipulator, changing from the initial singular configuration to the final singular configuration with the corresponding rank
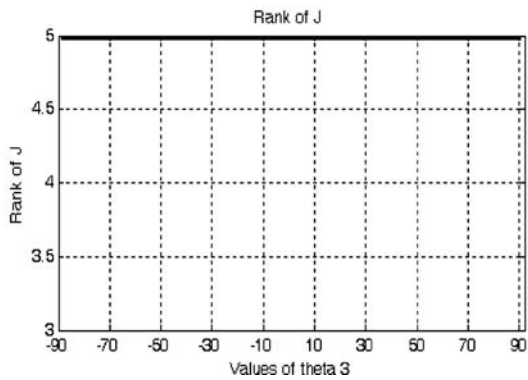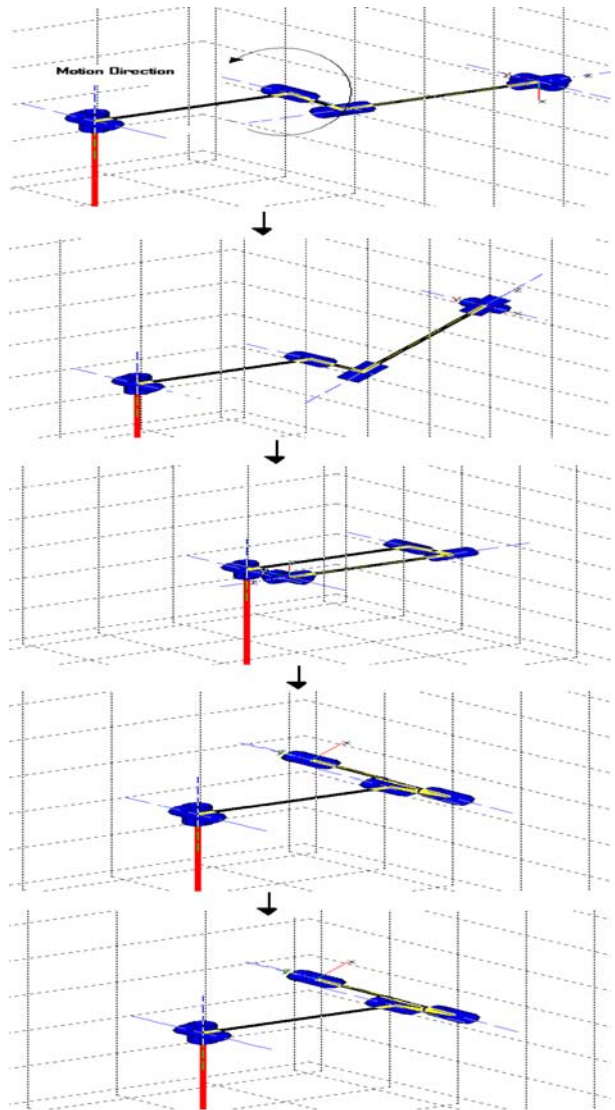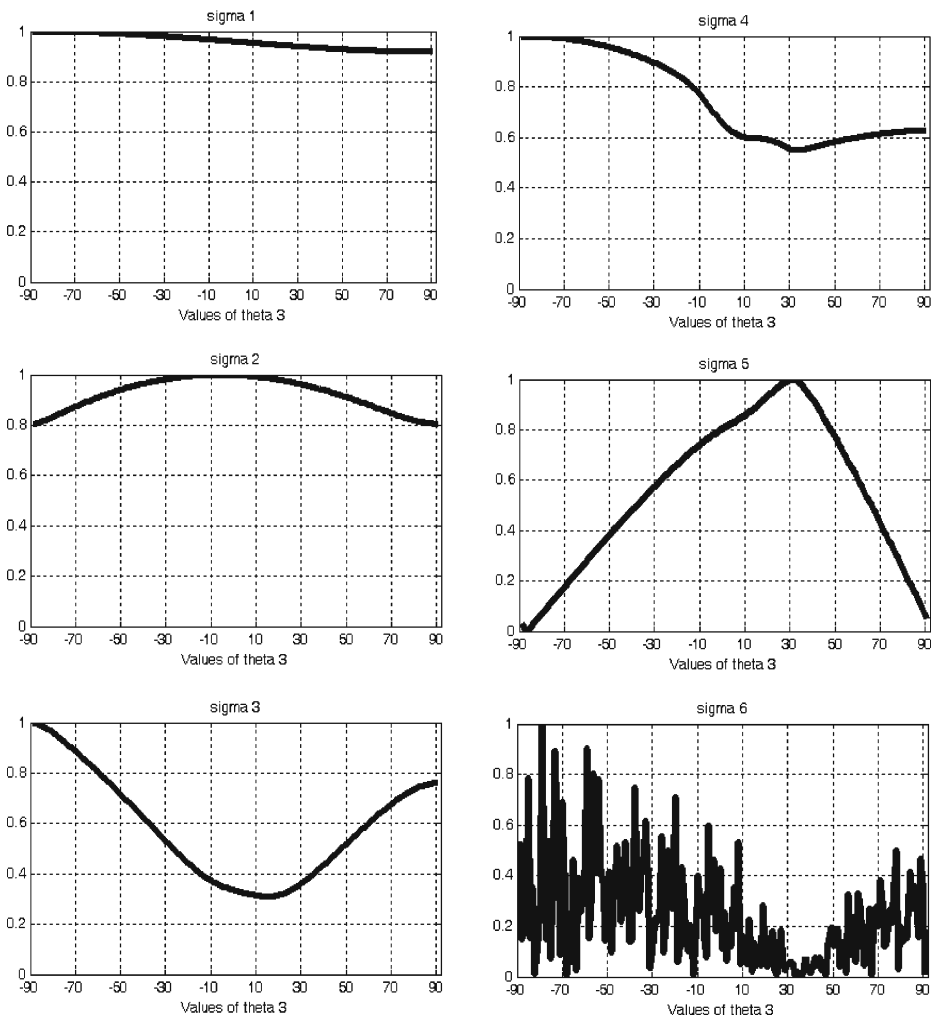
**Fig. 2** The behavior of σ_1 to σ_6 during the experiment

4 Experiments

In this section, we will show and explain some results using serial manipulators with D-H parameters illustrated in Tables 1, 2 and 3. We have proposed some assumptions which can be summarized as follows:

- In our case study, we have dealt with the arm manipulability regardless of the orientation singularity.
- We study non redundant manipulators only.

Fig. 3 Manipulability's bands of the puma 560 manipulator in 2D workspace according to σ_5
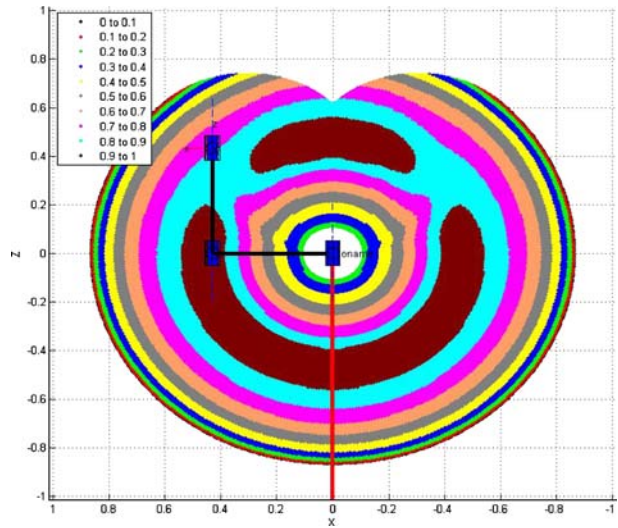

4.1 The Puma 560 Manipulator

In the Puma 560, we have experienced that the third joint is the cause of singularity. The sample trajectory of this manipulator is from the initial position $Q_{initial} = [0, 0, -\frac{\pi}{2}, 0, 0, 0]$ to the final position $Q_{final} = [0, 0, \frac{\pi}{2}, 0, 0, 0]$ is shown in Fig. 1. The DH parameters of the Puma 560 are shown in Table 1.

**Fig. 4** The behavior of σ_3 during the experiment
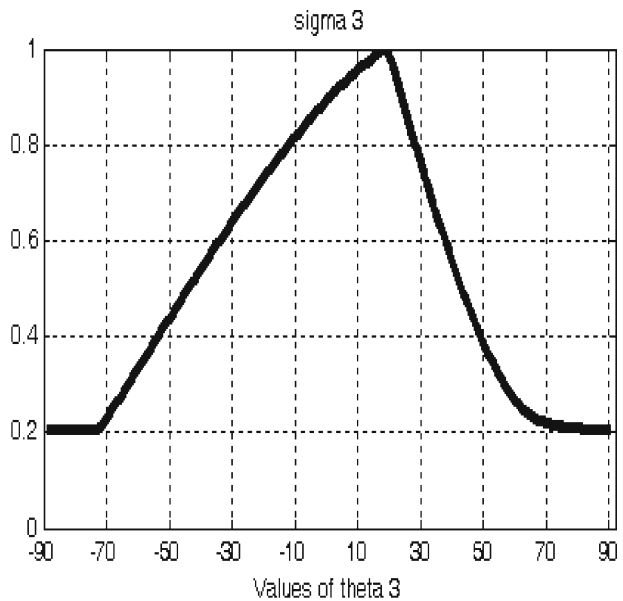
Fig. 5 Manipulability's bands of a six degrees of freedom manipulator 2-D
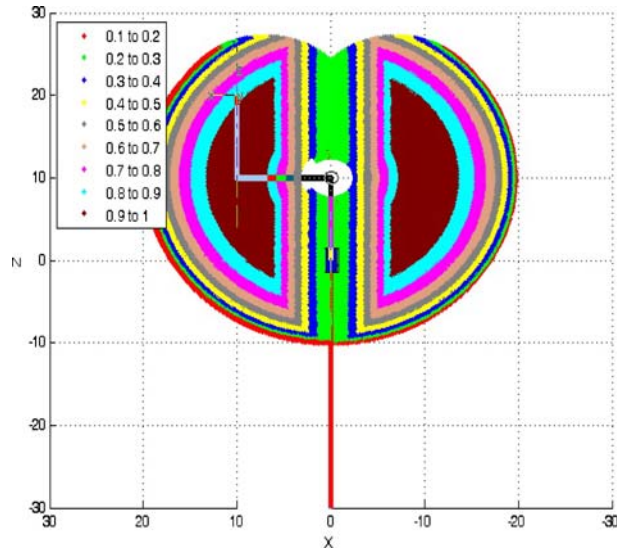

In Fig. 2, it is obvious that σ_5 is exactly expressing the manipulability's index. Furthermore, the rank of the Jacobian matrix during this experiment was constant at 5 because joint 6 and joint 4 were on same axis during the whole experiment. The manipulability index of every point within the whole workspace is represented in bands and each band is visualized using a different color as shown in Fig. 3.

Figure 3 is considered important in our research strategy since it provides a visual demonstration for the manipulability measure for the entire workspace for the Puma

Fig. 6 The behavior of σ_3 during the experiment
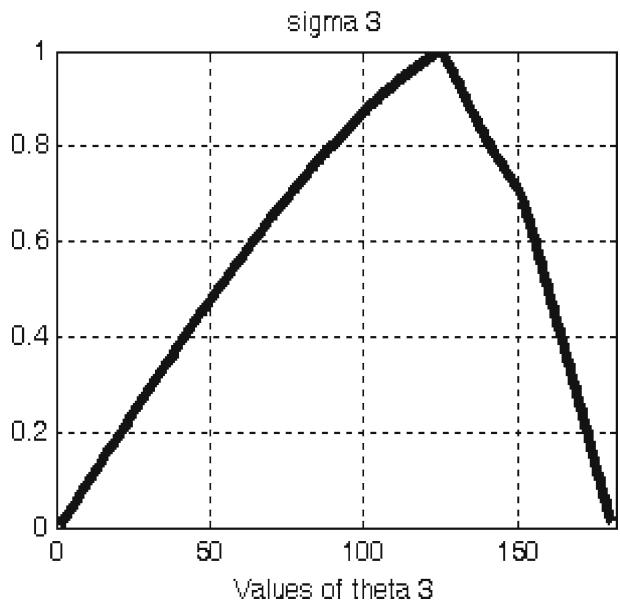
Fig. 7 Manipulability's bands of the Mitsubishi Movemaster manipulator in 2-D workspace
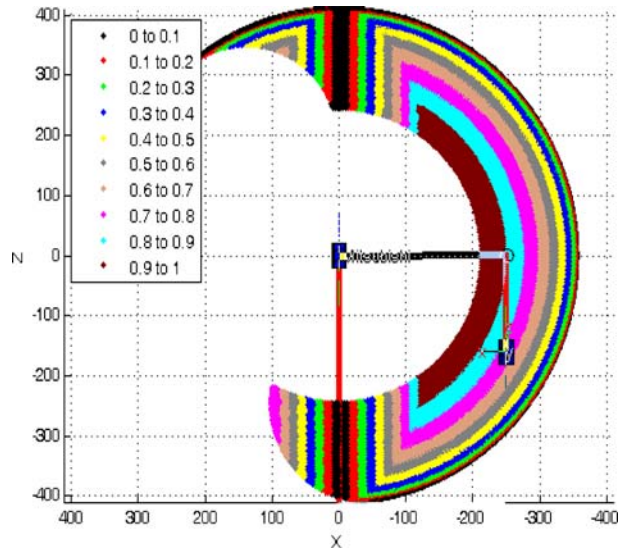
560 manipulator. These results can strongly contribute in developing an intelligent mobile manipulator. For example, the positions with the highest manipulability index will have better dexterity compared with those of the lowest manipulability index.

4.2 A Six Degrees of Freedom Serial Manipulator

Similarly, we implemented the same procedure for the Mitsubishi Movemaster manipulator and a regular six DOF manipulator. In a six DOF manipulator, the sample trajectory of this manipulator is from the initial position $Q_{initial}= [0, 0, -\frac{\pi}{2}, 0, 0, 0]$ to the final position $Q_{final}= [0, 0, \frac{\pi}{2}, 0, 0, 0]$. The DH parameters of this manipulator are shown in Table 2. The behavior of σ_3 during the experiment is shown in Fig. 4. The visual demonstration of the manipulability index is shown in Fig. 5.

4.3 The Mitsubishi Movemaster Manipulator

The initial position is $Q_{initial}= [0, 0, -\frac{\pi}{2}, 0, 0, 0]$ and the final position $Q_{final}= [0, 0, \frac{\pi}{2}, 0, 0, 0]$. The DH parameters of this manipulator are shown in Table 3. The behavior of σ_3 during the experiment is shown in Fig. 6. The visual demonstration of the manipulability index is shown in Fig. 7.

Table 4 Summary of results

Manipulator	Order of σ expressing the manipulability	Min rank of the Jacobian matrix
Puma 560	5	5
Six DOF	3	3
Mitsubishi Movemaster	3	3

4.4 Experimental Results

It is obvious from Table 4 that we can suppose that the order of σ that is expressing the kinematics manipulability's index equals to the minimum rank of the Jacobian matrix.

5 Conclusions and Future Work

In this paper, we present a new algorithm for measuring manipulability, and then we implement simulations supporting our methodology on different manipulators. The manipulability measure is crucial in performing intelligent behavior tasks such as grasping, pulling or pushing objects with sufficient dexterity.

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