New Concept in Optimizing Manipulability Index of Serial Manipulators, Using SVD Method

Mohammed Magdy[1], Ayssam ElKady[2,] and Tarek Sobh [2]

1- Mechanical Engineering Department
2-Computer Science Department
University of Bridgeport, Bridgeport, CT 06604 USA
{Mohammem, Sobh, aelkady}@bridgeport.edu
2007

Abstract-**Studying the Manipulability index for every point within the workspace of any serial manipulator is considered one of the important problems, required for designing trajectories or avoiding singular configurations. We had to go through this problem for our project of optimizing D-H parameters of extending link/s for Mitsubishi which is existed at our lab. First step is to get all points within the workspace envelope with different manipulability bands and to find most the high band to start forward kinematic design and optimization of additional link/s parameters**.

We have discovered a new concept in this paper ,related for one method used before in measuring kinematic Manipulability .We have made simulations supporting our method on Puma560, Mitsubishi movemaster® , and six degrees of freedom manipulator.

I.Introduction

The benefit of using quantitative measure is well known in engineering systems. In other words, a quantitative measure provides one with a rational base which one can - without depending on experience -analyze, design and control the systems as follows: [1]

1-It helps one to evaluate performance of the system in simulation mode before involving in real time mode.

2-It helps one to design new system or at least modifying existing system from points or level providing high performance from this quantitative measure's view.σ

Manipulability measure is considered one of the most important quantitative measure in Robotic field .many mathematical methods have been proposed to describe robot's dexterity (Manipulability) and whether it can be optimized by observing the distance between singular configuration or not. Most of these previous researches have been tested while the manipulator is passing through line segment as robot's path from the basis of manipulator until last point on the robot's path, that manipulator can reach (at the boundary of the workspace envelope). The first points on path considered one with low dexterity (Manipulability) or even zero dexterity because in this configuration, one or more links are completely folded or at least are very close to this singular configuration. Also, when the manipulator reaches last point in the line segment, it also reaches another singular configuration, while it is fully extended.

Many mathematical formulations have been proposed to express the Manipulability measure; the product of all singular values of Jacobian, $\det (JJ^T)^{1/2}$, by Yoshikawa. The minimum singular value of the Jacobian, $\min (\sigma (J))$, by Klein and Blaho as a measure of the closeness of the manipulator postures to singularities. The reciprocal of the condition number, termed the conditioning index (CI), has been proposed by Angeles to measure the closeness of a configuration to singularity.[2]

As mentioned in [3], the determinant of jacobian can not be used for expressing Manipulability's index .It just becomes zero when a manipulator reaches any singular configuration. Other method has been proposed called reciprocal of the Jacobian as in [1,2,3] and we have tried to test this method with different concept and did not prove any efficiency for non linear path.

It is worth to mention that we are interested in arm Manipulability(that can be measure as distance from singularity) not manipulability of orientation, so we also tried to decouple the jacobian matrix according to [6] and the result , unfortunately was not satisfied at all ; bands were intersected with each others. Last method proposed was Singular Value Decomposition of jacobian matrix.

II.Singular Value Decomposition: [5]

Singular value decomposition is a mathematical technique to optimize the singular form of the any square or non square matrix $A_{(m \times n)}$, depending on eigen values, this is technique has been used in many engineering application.

$$A_{m \times n} = U_{m \times m} S_{m \times n} V^T_{n \times n} \quad (1)$$

$$U^T U = I \quad (2)$$

$$V^T V = I \quad \text{(i.e. U and V are orthogonal)} \quad (3)$$

T. Sobh et al. (eds.), *Novel Algorithms and Techniques in Telecommunications, Automation and Industrial Electronics,* 185–191.

$$S=\begin{pmatrix}\sigma_1 & 0 & \cdots & 0 & 0 & \cdots & 0\\ 0 & \sigma_2 & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & 0 & 0 & \cdots & 0\\ 0 & 0 & \cdots & \sigma_m & 0 & \cdots & 0\end{pmatrix} \quad (4)$$

We have constructed the jacobian according to [5]

$$J=[J_1\ J_2\ldots\ldots\ldots\ J_n]; \quad (5)$$

Where n is number of degrees of freedom

$$J_i=\begin{pmatrix}Z_{i-1}x(O_n-O_{i-1})\\ Z_{i-1}\end{pmatrix} \quad \text{For revolute joint (6)}$$

U and V are orthogonal matrix means U x U^T =1 & V x V^T = 1 and the singular values are in Descending orders $\sigma_1 \geq \sigma_2 \geq \ldots\ldots \geq \sigma_m$. Matrix A has full rank means rank of A= m when $\sigma_m \neq 0$. When $\sigma_m \approx 0$ with small tolerance the rank of matrix A looses one or more degrees of freedom and that means mathematically, the matrix looses one or more degree of freedom (one row or column becomes non independent ; it can be optimized by multiply another row or column by factor). Physically, any manipulator has two joint's axes coinciding on each other. For example, puma 560® at initial position; the sixth joint and fourth joint are on same axes so if the fourth or sixth joint moves, the result motion (orientation) will be the same. That means the manipulator looses one degree of freedom.

In past research, there was an argument about whether the minimum value of theses σs or the multiplication of all σs exactly represent the Manipulability's index.[4]

In this paper, we will propose a new concept for measure and observe this index plus make sure by drawing the bands of this index. Moreover, it will be discussed a new relationship between the minimum rank of the jacobian matrix, the manipulator can reach in any singular configuration (assuming robot's joint from [0 to 360 degree]), and the order of one of these σs that can exactly express the Manipulability's index.

III. NUMERICAL EXAMPLE

In case of singular configuration of Puma 560 at Q= [0, 0,-90, 0, 0, 0]

$$J=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ 20 & 0 & 0 & 0 & 0 & 0\\ 0 & 20 & 10 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 1\\ 0 & 1 & 1 & 0 & 1 & 0\\ 1 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$U=\begin{bmatrix}0 & 0 & 0 & 0 & 1 & -0.0034\\ 0 & -0.9988 & 0 & 0 & -0.0002 & -0.0499\\ -0.9982 & 0 & 0 & 0.0600 & 0 & 0\\ 0 & 0 & -1 & 0 & 0 & 0\\ 0.06 & 0 & 0 & 0.9982 & 0 & 0\\ 0 & -0.0499 & 0 & 0 & 0.0034 & 0.9987\end{bmatrix}$$

$$S=\begin{bmatrix}22.4010 & 0 & 0 & 0 & 0 & 0\\ 0 & 20.0250 & 0 & 0 & 0 & 0\\ 0 & 0 & 1.4142 & 0 & 0 & 0\\ 0 & 0 & 0 & 1.0935 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$V=\begin{bmatrix}0 & -1.0000 & 0 & 0 & 0 & 0\\ -0.8939 & 0 & 0 & 0.1852 & 0.4074 & -0.0270\\ -0.4483 & 0 & 0 & -0.3638 & -0.8147 & 0.0539\\ 0 & 0 & -0.7071 & 0 & -0.0467 & -0.7056\\ -0.0027 & 0 & 0 & -0.9129 & 0.4074 & -0.0270\\ 0 & 0 & -0.7071 & 0 & 0.0467 & 0.7056\end{bmatrix}$$

It is obvious that in singular matrix S; σ_5 and σ_6 are zero with small tolerance .That is because there are two singular cases in its Configuration; the forth and sixth joint are on same axis plus it is in singular arm configuration , so also σ_5 is zero .

TABLE 1
D-H Parameters of Puma 560[7]

i	α	A	Θ	D	Initial limit	Final limit	Joint's Type
1	90	0	*	0	-170	170	R
2	0	0.431800	*	0	-225	45	R
3	-90	0.020300	*	0.150050	-250	75	R
4	90	0	*	0.431800	-135	100	R
5	-90	0	*	0	-100	100	R
6	0	0	*	0	-180	180	R

TABLE 2
D-H Parameters of Six Degrees of freedom Serial Manipulator

i	α	A	Θ	D	Initial limit	Final limit	Joint's Type
1	90	0	*	10	Same joint's limit of Puma560		R
2	0	10	*	0			R
3	-90	0	*	0			R
4	90	0	*	10			R
5	-90	0	*	0			R
6	0	0	*	0			R

TABLE 3

D-H Parameters of Mitsubishi Movemaster®

i	α	A	Θ	D	Initial Limit	Final Limit	Joint's Type
1	90	0	*	300	-150	150	R(Pos.)
2	0	250	*	0	100	130	R(Pos.)
3	0	160	*	0	-110	0	R(Pos.)
4	-90	0	*	0	-90	90	R (Orient.)
5	0	0	*	72	0	0	R (Orient.)

IV. Assumptions

Our assumptions can be mentioned in two points;

1- We are interested in (arm) Manipulability, regardless orientation singularity.
2- We are interested in studying the non redundant Manipulator in x-y-z plane.

V. Simulation Strategy

In our research, we tried to test singular value of any serial manipulator according to these steps:
1- Finding the joint that has a vital role in changing the serial manipulator's configuration to singular one.
2- With simulation program; using robotic toolbox [7] making this joint ranging from one value to the other value.
3- Calculating the jacobian in every step in loop and Singular matrix.
4- Plotting every σ (normalized) and also the rank of the jacobian with number of configurations, robot has passed through changing from one singular phase to another. See fig (1)

$$\text{Normalized}\,\sigma_i = \frac{\sigma_i}{Max\{\sigma_{i1},\sigma_{i2},\sigma_{i3},.....,\sigma_{in}\}} \qquad (7)$$

i: Order of σ in the singular Matrix.
n: number of loops/ configurations during simulation.

5-Checking the rank of Jacobian while the serial robot in any other different configurations.

A Experimental Results for Puma560

The initial position is $Q_{initial}$ = [0, 0,-90, 0, 0, 0] degree and the final position is Q_{final} = [0, 0, 90, 0, 0, 0] degree.

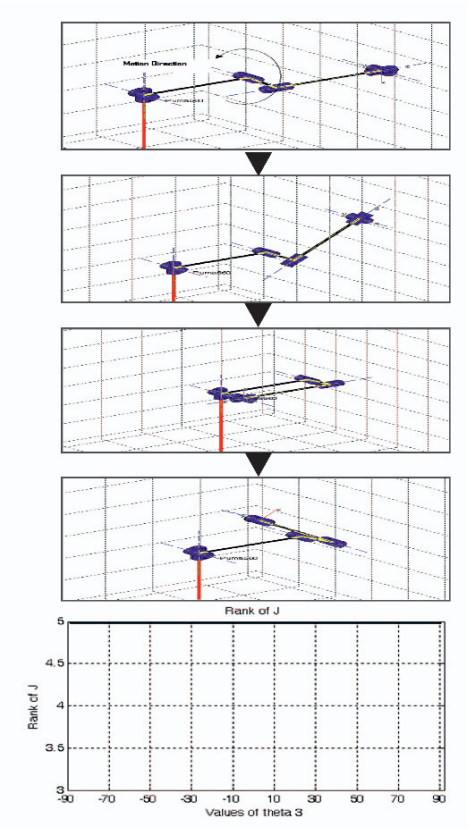


Fig1. Phases of Puma 560 in our experiment changing from one singular configuration to the other.

In Fig.2-a and fig 2-b, It is obvious that σ5 is exactly expressing the Manipulability's index. Also, rank of the jacobian during the experiment was constant at 5 (joint 6 and joint 4 were on same axis during the experiment).
In Fig.3, We have used another algorithm, depending on forward kinematic to generate the whole workspace of any manipulator in 2-D. Algorithm is:
1- Forward Kinematics
2- With every point, calculate Jacobian matrix.
3- Calculate SVD
4- Record in a matrix [Qs x y z σ_K]; K=min (rank (J)) [in all singular configuration, assuming every joint has a range from 0 to 360 Degree].
5- Plotting the coordinates with different colors according to σ_5 see fig (3).

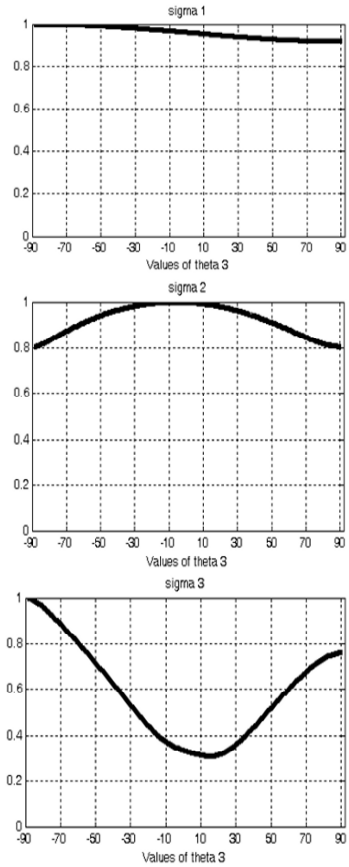


Fig. 2-a. The Behavior of **σ** 1 to **σ** 3 during the experiment

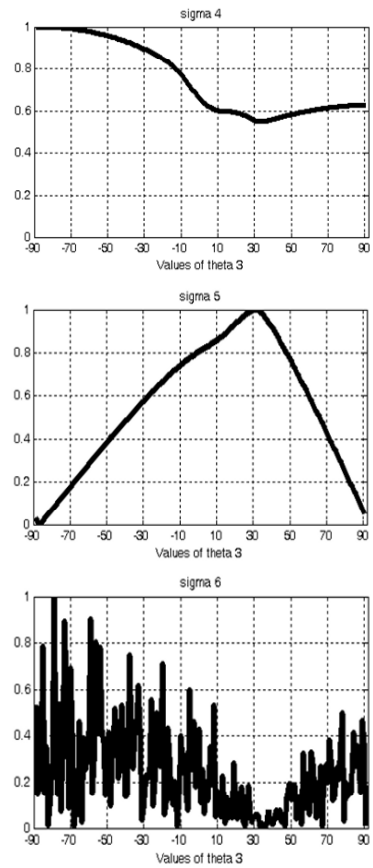


Fig. 2-b. The Behavior of **σ** 4 to σ 6 during the experiment

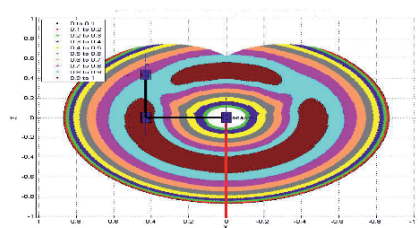


Fig.3. Manipulability's Bands of Puma 560 in 2-D workspace according to $\boldsymbol{\sigma_5}$

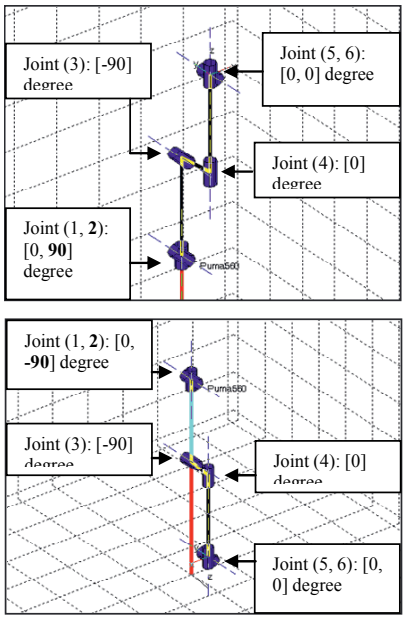


Fig.4. Two other Singular Configurations of Puma560 manipulator

Similarly, we did same strategy for Misubishi Movemaster® and Regular six degrees of freedom manipulator.

B Experimental Results of Six degrees of freedom Serial Manipulator

The initial position is $Q_{initial}$ = [0, 0,-90, 0, 0, 0] degree and the final position is Q_{final} = [0, 0, 90, 0, 0, 0] degree.

It is obvious from fig.6 that σ_3 is the only singular value in singular matrix that expressing exactly the arm Manipulability (fig.7 is supporting that by showing Manipulability bands perfectly).We have also observed the rank of the jacobian matrix during these phases was five when the manipulator was not in singular value and was four when the manipulator in singular value. Also, we have checked the rank of jacobian in other singular configuration as in fig.8. It was a surprise that the rank of the jacobian matrix has been reduced to three.

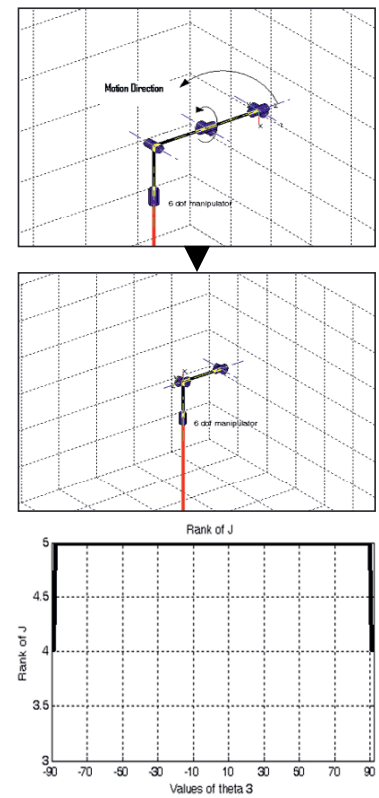


Fig.5. Phases of sex degrees of freedom manipulator in our experiment changing from one singular configuration to the other

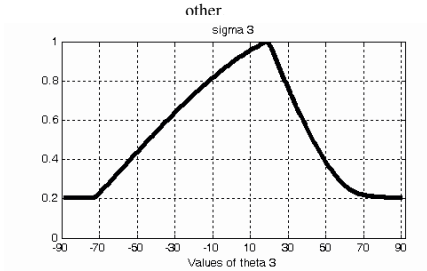


Fig.6. the Behavior of σ_3 during the experiment

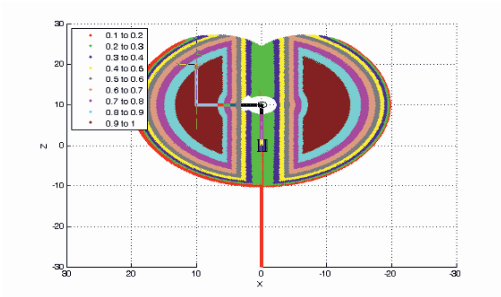
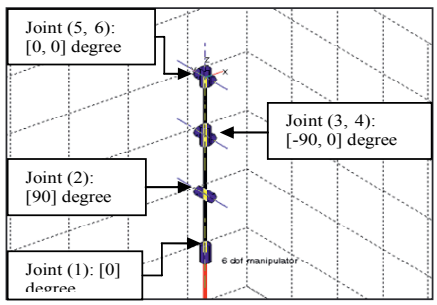


Fig.7. Manipulability's Bands of six degrees of freedom manipulator 2-D



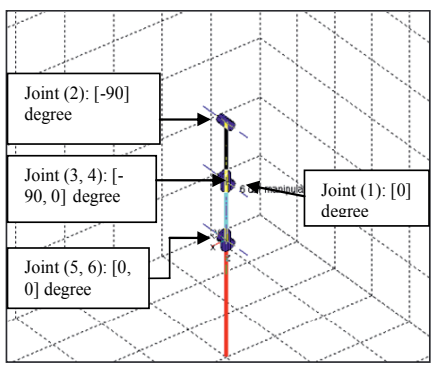


Fig.8. Two other Singular Configurations of six degrees of freedom manipulator

C Experimental Results Mitsubishi Movemaster ®

The initial position is $Q_{initial} = [0, 0, 0, 90, 0]$ degree and the final position is $Q_{final} = [0, 0, 180, 90, 0]$ degree.

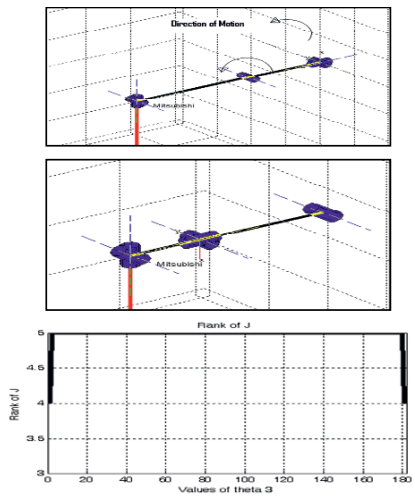


Fig.9. Phases of Mitsubishi Movemaster® in our experiment changing from one singular configuration to the other.

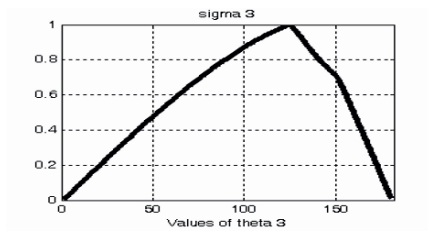


Fig.10. the Behavior of σ3 during the experiment

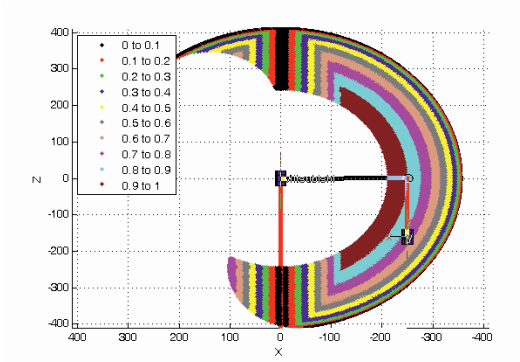


Fig: 11. Manipulability's Bands of Mitsubishi movemaster in 2-D workspace.

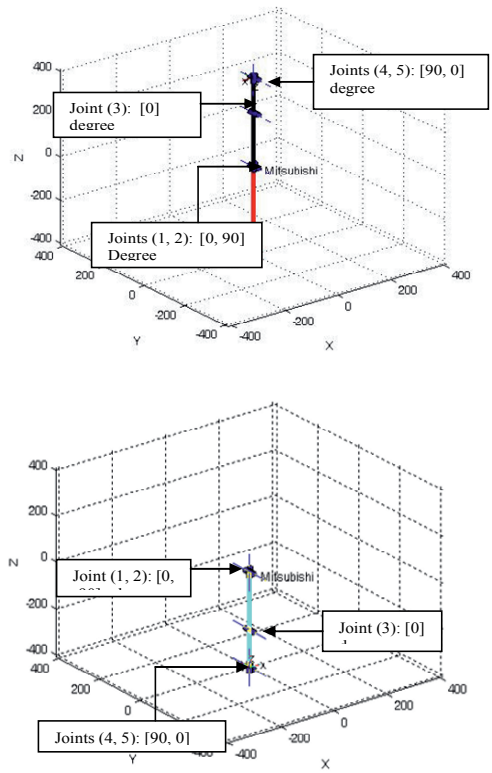


Fig.12. Two other Singular Configurations of Mitsubishi manipulator

TABLE 4
Summary of Results

Types Of Manipulator has been used in the Experiment	Order of σ that is expressing the Manipulability	Rank during changing the configuration from one singular to another	The least rank can robot reach in any other singular configurations
Puma560®	5	5	5
Six degrees of freedom	3	5 and 4	3
Mitsubishi Movemaster®	3	5 and 4	3

IX. Conclusion

It is obvious from the table 4 that the order of σ that expresses kinematic manipulability 's index or bands equals to the minimum rank of the jacobian matrix, that robot can reach in any singular Configurations , regardless the joints' limits.

X. Future Work

1- We will discuss in the future how to prove this relation mathematically.
2- As we mentioned above we have to optimize all points with deferent Manipulability's index to find max value of one of the σs and normalize to make the index's values ranges from zero to one. We will try to optimize the jacobian symbolically, and optimize the singular value decomposition also symbolically to find the differentiation of this value to optimize the maximum value.
3- What is the best location to fix a new robot's base already has a task to cover all points on trajectory with high Manipulability's index for every point. For instance, it was in past big argument about what is the best location to fix a manipulator will be used for painting the F-116 fighting plane.
4- Actually, we are thinking about new concept, we have not called yet, about resultant Manipulability between two serial manipulators, interacting with each other to do one task. For instance, one is holding the work piece and the other is doing some machining or welding operations.
5- Same concept can be taken into consideration ,when two manipulators are fixed on two different mobile robots or prismatic joint, holding a spherical or difficult symmetric object for instance, ball .From Kinematic Manipulability , what is the best posture with high Manipulability far enough from singular configuration to hold this object?
6- Now, kinematic manipulability becomes mature enough to be involved in any commercial software for robot simulation such as: Delmmia ®, Easy Rob ®..Etc

References

[1] P. H. Chang, "A Dexterity Measure for the kinematic Control of Robot Manipulators with Redundancy", February, 1988.
[2] I-Min Chen, "theory and application of modular reconfigurable robotic systems", 1994.
[3] T. Tanev, and I. Chandrov, " Performance evaluation of manipulation system and graphical representation of the characteristics", 1998.
[4] T. Tanev, and B. Stoyanov, "On the Performance Indexes for Robot Manipulators", 2000.
[5] H. Bruyninckx," The Robotics WEBook" , 2006.
[6] M. W. Spong , S. Hutchinson , and M. Vidyasagar, "Robot modeling and control", 2005.
[7] P. I.Corke, "Robotic toolbox", April 2002.