

# Review of Neurobiological Based Mobile Robot Navigation System Research Performed Since 2000

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**Abstract**— In an attempt to better understand how the navigation part of the brain works, and to possibly create smarter and more reliable navigation systems, many papers have been written in the field of bio-mimetic systems. This paper presents a literature survey of state of the art research performed since the year 2000 on rodent neuro-biological and -physiological based navigation systems that incorporate models of spatial awareness and navigation brain cells. The main focus is to explore the functionality of the cognitive maps developed in these mobile robot systems with respect to route planning, as well as a discussion/analysis of the computational complexity required to scale these systems.

## 1 INTRODUCTION

This paper reviews the current state of research in mobile robot navigation systems that are based on the rodent's specialized spatial awareness and navigation brain cells. Specifically, these cells include: place cells, grid cells, border cells and head direction cells. The advantages of using a neurobiological based system includes the possible performance rewards that may be realized in the future pertaining to navigation and smart systems, as well as the benefits of using accurate models of the brain for other, related research [1, 2]. For artificial intelligence to take a major leap forward, machines will at minimum need to learn and think the way humans do. This will require computational elements that behave similar to, and are as compact as, the neurons, and accompanying dendrites and axons found in the human brain.

Although there is a need for new technical paradigms in artificial intelligence, this paper doesn't propose or present new methods, but outlines work that may be a path to such answers. The most important attributes of the neurobiology based navigation systems covered, are the types of cognitive maps produced by these systems and how they are, or can be, used for route planning. Thus, the focus of the analysis of the reviewed literature will be centered on mapping and route planning capabilities of these neurobiological based systems.

Most papers that have reviewed such systems came around or before the year 2000, such as [3-5]. However, since 2000, much advancement has taken place in the miniaturization of electronic packages (Moore's Law), thus increasing the practicality of placing better sensors, processors, algorithms, more memory, etc. on mobile robots.

In addition, the discovery of the grid cell by the Mosers in 2005 [6-9] added more insight as to how rodents navigate. Therefore, this paper fills the gap on a needed formal review of the state of the art neurobiological based navigation systems researched and developed from 2000 and on. On the non-neurobiological (classical) side of navigation in mobile robots, a good source that reviews map-learning and path planning strategies can be found in a paper by Meyer and Filliat, 2003 [10], as well as many books on the topic (e.g., [11]).

Thus, the outline of this survey proceeds as follows: Section II discusses the basics of the simultaneous localization and mapping (SLAM) method of navigation, as well as some fundamental issues that plague every navigation system (neurobiology based or not); Section III gives a brief review of the definitions of the neural cells that will be the center of focus in this paper; Section IV covers state of the art research that has been performed on neurobiological based navigation systems (only those that have been realized in working, prototype mobile robot systems) with a critique of the cognitive maps developed for route planning algorithms at the end of each subsection; and Section V presents an analysis of the scalability of these systems to incorporate more neurobiological based features with respect to the adequacy of the computational resources that can fit on a mobile robot for neural network processing.

## 2 GENERAL ROBOT NAVIGATION BACKGROUND

### 2.1 Simultaneous Localization and Mapping

For a mobile robot to be truly autonomous, it needs to be able to operate and navigate without human intervention and in a non-specially engineered environment. More specifically, the following needs to be true: A mobile robot must be able to locate itself in an unknown location, of an unknown environment by incrementally building a map of its environment, while simultaneously locating itself in that environment by use of the derived map. This process is known as simultaneous localization and mapping (SLAM) [12-15]. As described in [15], the fundamental parts of a classical SLAM system are: (1) landmark extraction, (2) data association, (3) state estimation, (4) state update, and (5) landmark update. Of course, to be able to accomplish these SLAM steps the system requires hardware, used by the agent to interact with the environment and make decisions with (i.e., sensors, actuators, processor, etc.), plus any filters and/or methods required to adequately perform these 5 tasks (e.g., sensor noise suppression, error correction algorithms, etc.). SLAM is not unique to just classical systems. It is accomplished, in some similar form, by rodents by use of their hippocampus [2, 16-18]. The special neurons or brain cells which accomplish this will be covered in Section 3.

## 2.2 Fundamental Navigation Issue: Sensor Error

There are fundamental issues which plague every navigation system (neurobiological based or not) [19]. These issues, largely path integration related, propagate up into the mapping and localization phases, levels L0 to L1 in Fig. 1. In a neurobiological or neurophysiological based navigation system, this is equivalent to either lesions introduced into hippocampus and related areas, lack of allothetic stimuli, or other similar targeted manipulations on rats [9, 20, 21]. The outcome, thus, has a negative effect on the accuracy of the overall navigation system.

Therefore, for any navigation system to work adequately, the mobile robot's sensor data error must be within a usable margin and be reset periodically by allothetic information, whether visual, tactile, olfactory or other. Idiopathic data is the most basic navigational data for the robot to use to track its movements and is the basis for path integration [22-24]. The inherent issue with any ground based robot navigation system is the mobile robot's measurement accuracy with respect to distance traveled and directionality (idiothetic data), since this data is used to derive the robot's or rodent's pose. Sources of classical and most neurobiological based navigation systems' measurement errors come from the data obtained from odometry devices, inertial measurement units (IMUs), distance sensors, and other position/pose measurement systems (use of idiothetic stimuli only). The source of these errors fall into two categories, as described in [24, 25], of being either systematic or nonsystematic. Additionally, these errors accumulate over time [15, 22, 24-26], making environment localization and mapping inaccurate if these measurements are used directly. Methods in error correction of odometry and related position/pose data has been, and still is, a major topic of research. However, probabilistic filters (e.g., extended Kalman filter - EKF) or particle filters, as well as use of allothetic stimuli (e.g., landmarks), are used with SLAM algorithms in classical systems to help correct these errors in the pose data and location estimation.

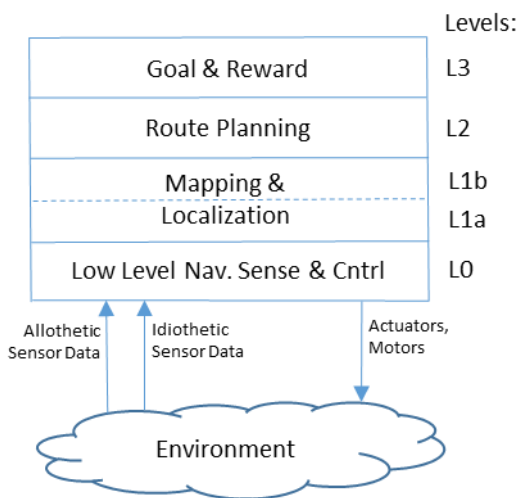


Fig. 1: A generic SLAM based hierarchal architecture that can apply to both classical and neurobiological based mobile robot systems.

Similarly, whether animals, insects or animats, these navigation systems require path integration (PI) systems with

corrective error mechanisms [6, 20, 27]. In the case of animats, or more specifically, the neurobiological based navigation systems reviewed in this paper, it is shown that visual data is key to keeping PI errors to a workable minimum. This will also be touched on in Section 5.

## 3 NAVIGATION RELATED CELLS REVIEW

The following is a review of the definitions and characteristics of the specialized navigation neurons or cells, as found in the hippocampus and entorhinal cortex of a rodent brain, as well as the human brain [28]. This material is covered in other literature [6, 17, 18, 29, 30], but is included here for completeness.

### 3.1 Place Cells

Place cells in rodents were discovered by O'Keefe and Dostrovsky in 1971 [31, 32]. These cells, primarily located in the CA1-CA3 regions of the hippocampus, each fire at a devoted location in of a rodent's roaming area. The place cell's firing location is invariant to the head direction or body pose of the rodent. The firing area of each place cell also seems to follow the summation of two or more Gaussian distribution curves, one for each salient distal cue [28].

### 3.2 Head Direction Cells

Head direction cells were discovered in rodents more than a decade after the place cells [33, 34]. These cells are place invariant and each have a preferred direction with respect to the rat's head direction in the horizontal plane, where it will fire at a maximum rate. They are silent for all other directions, except for a small region ( $\pm$  a few degrees) of their preferred direction angle. The head direction cells only fire as a function of the rat's head direction and not its body. Additionally, although the cells have different preferred directions, they seem to fall into a finite set of directions (e.g., N, NE, and SW). The directionality is relative, such that they will align relative to a dominate external cue of the environment the rat is introduced to, if available, else it will set a direction based on other unknown origins [20, 35].

### 3.3 Border Cells

A border cell can be thought of as a specialized place cell, where it only fires with respect to a certain border or barrier [36, 37]. The area covered by a border cell can vary drastically, with respect to each other. Similar to the place cell, the firing characteristic of the border cell is invariant to the rat's head direction.

### 3.4 Grid Cells

The grid cell was discovered by Edvard and May-Britt Moser in 2005 [6-9]. This set of special, navigation related brain cells, which is the most recent to be discovered, is located in the entorhinal cortex. Grid cells have a very interesting firing characteristic, as compared to the places and border cells. A single place cell and border cell only fire at a specific location/region, whereas a single grid cell fires at a geometrical constellation of locations/regions. These regions within the rodent's roaming area form

hexagonal/equilateral triangles. Thus, once a rodent has learned any spot in the locations that are covered by a single grid cell, it will fire every time it passes over the same learned spots of this grid (within some non-symmetrical region of this location).

#### 4 STATE OF ART RESEARCH IN NEUROBIOLOGICAL BASED NAVIGATION SYSTEMS FOR MOBILE ROBOTS

This section covers state of the art research in neurobiological based navigation systems, where the systems have been implemented in a mobile robot since the early 2000's. These systems fall into three categories, based on the centric navigation cell that is being functionally emulated. These categories are: place cell centric, theoretical cell centric and grid cell centric. The theoretical cell uses one or more true neural navigation cells (one being the place cell typically) to create a new, fictional cell that is at the center of its navigation systems. Although fictional, these cells, or functions, may indeed be plausible and real in one form or another. Basic features and capabilities of these systems are summarized in Table I, located at the end of the paper.

##### 4.1 Place Cell Centric Systems

###### 4.1.1 Arleo & Gerstner '00a

The article and study by Arleo and Gerstner, 2000a [38] has had an influence, in one form or another, on many future works covered in this section, particularly [2, 39]. The references used in [38] fall into the categories of both neuroscience: O'Keefe & Nadel, 1978 [40]; Taube et al., 1990 [33]; Redish, 1997 [4]; etc.; and in neurobiological inspired circuits and models: Burgess et al., 1994 [21]; Brown & Sharp, 1995 [41]; Redish & Touretzky, 1997 [42], Gaussier et al, 1997 [43]; etc., which form a basis of references used by the other proceeding studies/articles. More references can be found in Arleo & Gerstner, 2000a [38] & 2000b [44]. Additionally, this paper's presentation and functional use of neurobiological specialized spatial navigation cells found in the rodent's hippocampus, for modeling in robotic navigation, is central to the theme of all of these papers covered.

###### 4.1.1.1 Head Direction and Place Cells for Spatial Navigation

In [38], the Khepera robot system used consists of: an on-board camera for vision based self-localization (90° field of view in horizontal plane), eight infrared (IR) sensors for obstacle detection and light detection, a light-detector for measuring ambient light, and an odometer for sensing self-motion signals. The neurobiological based navigation system models two crucial spatial navigation cells: head direction cells and place cells.

In Fig. 2, the allothetic inputs consist of data from the on-board camera, which is used for the place cells in the sEC submodule, as well as data from the eight IR sensors and the ambient light sensor, which are used by the visual bearing cells in the VIS submodule (left side of Fig. 2). The neural networks (Sanger's [45]) to the place cell from the camera

input, are programmed off-line during an initial unsupervised, Hebbian learning phase [46]. During this initial, exploration/neural network training phase, each place cell location is learned by dividing images taken into smaller 32 x 32 pixels, running the reduced image through 10 different visual filters of 5 set scales each. This is done for the North, West, South and East views of the robot's arena from each snapshot/place cell location. The weights for the neural networks of each cell are trained with the reduced images and adjusted for maximum response for each image location. Thus the place cells are programmed neural networks with the on-board camera image, divided into four quadrants of 32 x 32 pixels each, at the input, and will allow for self-localization in the online mode.

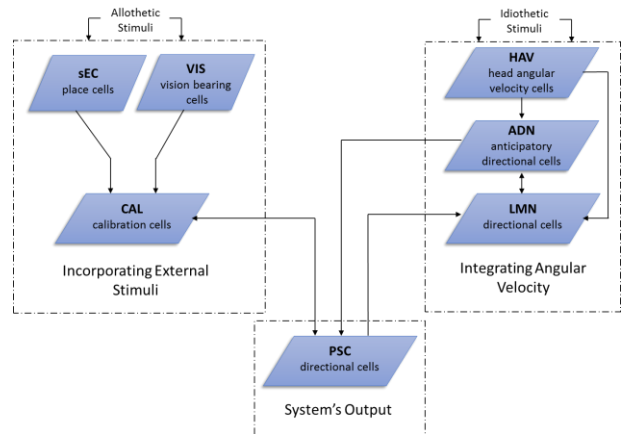


Fig. 2: A functional overview of the directional system [38].

A light source is added to one wall of the robot's arena, where the IR sensors and ambient light sensor can lock onto this global direction (with the help of neural networks for fine tune positioning to the light source). This allows for calibration of the robot's directional module (right side of Fig. 2), which bounds the accumulated error in directionality.

The robot uses three different neural populations of cells (right side of Fig. 2) to calculate its head direction from its: current angular velocity, anticipated angular velocity, and feedback from the system output and calibration cells. The end result is a set of quantized, directional cells to drive the robot's motors for proper heading.

###### 4.1.1.2 Computational Complexity

The computational complexity of this system is a bit more involved than briefly covered here. Further details can be found in [38, 44, 47]. However, any neural network system is going to have a relatively high to extremely high computational complexity, based on the number of neural networks and the processing status of off-line and on-line/real-time learning. The environment is somewhat engineered and needs to be static. This is true though of any system in the initial stages of wringing out system integration errors, model problems/accuracy, etc.

###### 4.1.1.3 Mapping and Route Planning

Visual based mapping, through the use of snapshot recognition (place cells), is used to help correct head

direction error and not for obstacle avoidance or route planning. Therefore, true mapping and any form of route planning are not addressed in [38] and [44].

### 4.1.2 Hafner '08

#### 4.1.2.1 Place Cells and Cognitive Maps

In [48], Hafner uses place cells for creating a cognitive map of a mobile robot's area. The mobile robot, outfitted with only an omnidirectional camera and a compass, produces a cognitive map during an exploration phase, where the map is represented by place fields and place cells. Each snapshot taken by the camera is converted into a 16-dimensional transformation, which is used as the sensory input to a neural network system. That is, each 360° camera snapshot is divided up into 16 angular, azimuth sections of 22.5° each, filtered and sent to the place cells' neural networks. The weights of each neural network, initially set to random values, take on evolved values during the exploration phase. The place cells, as shown in the "output layer/map layer" in Fig. 3, become relationally connected to each other based on a self-organizing map (SOM) methodology [49], where each, single winner of a particular snapshot becomes connected to the previous winner and the corresponding connection weight is increased. Since the place cells are not geometrically fixed, they are assigned relative angles to each other, creating a topological map.

#### 4.1.2.2 Simulated Route Planning

However, once the neural cognitive maps have been built, they can only be used in simulation for navigation. The topological and metric information requires too much memory to reside in the mobile robot. Thus the mobile robot relies on landmark (snapshot) recognition and use of the SOM to reach goal spots or areas.

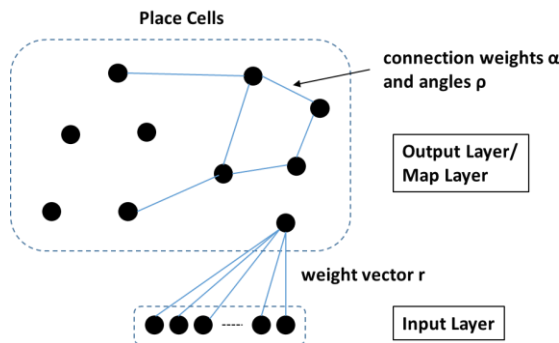


Fig. 3: Neural network structure as a result of learning connectivity between place cells. The input layer represents input from the robot's sensors [48].

### 4.1.3 Barrera and Weitzenfeld '08

#### 4.1.3.1 System Overview

Barrera and Weitzenfeld [2, 50] propose and implement a very complex, intricate and modular neurophysiological based navigation model. As with Arleo [38, 44], all of the proposed functionalities are mapped back to existing neurophysiological entities. Additionally, many of these modules are implemented using Gaussian distributions and

the Hebbian learning rule/equation for neural networks. The main goals of this research are: (1) for the mobile robot to be able to learn and unlearn path selections for goal locations based on changing rewards, (2) to create a realistic neuroscience based test bed for use in further behavior studies, and (3) add to the existing gap in the SLAM model between mapping and map exploitation [2]. The mobile robot's test environment configurations are limited to the T-maze and the 8-arm radial maze.

The neurophysiological theory that forms the basis for this study comes from [51]. Thus, in addition to idiothetic and allothetic sensory inputs, there are also internal state/incentives and affordances information sensory inputs. Fig. 4 shows the functional modules of this system, while removing much of the underlying details of the neurophysiological framework. Further details, such as model description, the neurophysiological framework, and equations for each of these modules can be found in [2, 52-54].

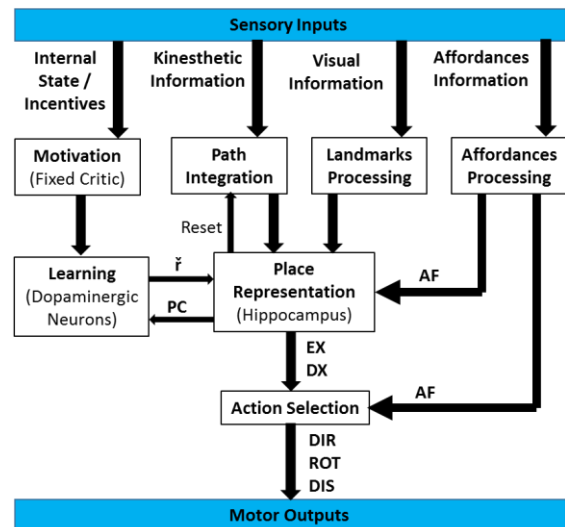


Fig. 4: Computational spatial cognitive model of the Barrera and Weitzenfeld neurophysiological based mobile robot navigation system. Some submodules and neurophysiological framework are not shown and can be found in [50]. Partial glossary:  $\dot{r}$  = effective reinforcement; PC = place information pattern; EX = expectations of maximum reward on their corresponding directions (DX); DIR = next rat direction; ROT = rat rotation; and DIS = next rat moving displacement.

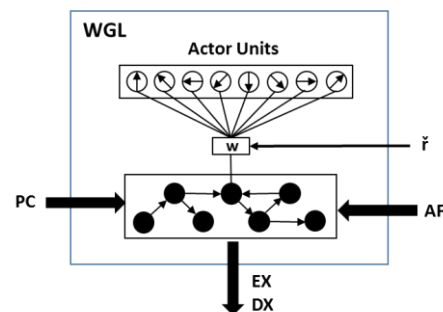


Fig. 5: World graph layer module which implements a topological map of the mobile robot's environment inside the Place Representation module.

Since the system lacks odometry and compass sensors, the idiothetic data comes in the form of kinesthetic data that is sent to an external motor control module, via the Action Selection module as shown in Fig. 4, which is used for executing rotations and translations of the robot.

#### 4.1.3.2 Place Cells, Cognitive Map Generation

The Place Representation module in Fig. 4 is where the cognitive map is made, stored and accessed for the mobile robot to select movement options. Thus, this module represents the functionality of the hippocampus. The path integration information is combined with landmark information, through the Hebbian learning rule, to create a place cell layer. The overlapping place cell fields in this layer represents given locations or nodes that are found in the world graph layer (WGL), as shown in Fig. 5.

The WGL uses a simple algorithm to decide its next move. It analyzes active nodes connected to the Actor Unit and based on the highest weight, the WGL chooses the step that will get it closer to its learned goal, or the best move for the time if a goal has been changed or not learned yet.

#### 4.1.3.3 Computational complexity

Because of the high computational complexity of this neurophysiological based navigation system, most of the model runs on an external 1.8 GHz Pentium 4 PC, which communicates wirelessly with a Sony AIBO ERS-210 4-legged robot. Thus, the system is not autonomous.

## 4.2 Theoretical Cell Centric Systems

### 4.2.1 Wyeth and Milford: RatSLAM, version 3

#### 4.2.1.1 System Overview

Wyeth and Milford focuses in [16, 17] on a neurobiological inspired, SLAM based, mapping system for a mobile robot navigation system, based on models and earlier versions of RatSLAM [40]. Their robot, a Pioneer 2-DXE base system, performs mock deliveries in a large, single floor, office building using simple sensors: motor encoders for odometry, sonar and laser range finder for collision avoidance and pathway centering, and a panoramic camera system for landmark recognition. This system, named RatSLAM, uses the concept of place cells coupled to head direction (HD) cells to derive, what they call, pose cells.

#### 4.2.1.2 Pose Cells

The competitive attractor network (CAN) [4, 40] based pose cells are used with local view cells, which are snapshots of the panoramic camera along the robot's journey. Thus, Milford and Wyeth, have added a new type of cell: the pose cell. The pose cell is similar to the conjunctive grid cells, as they report, which is a combination of grid cells and head direction cells found in the rodent brain. The pose cells work like weighted probabilities that each local view cell is in the direction and location of the stored pose (averaged). Fig. 6 illustrates the connectivity of the RatSLAM, version 3, as described here and in [20].

#### 4.2.1.3 Cognitive Map

The mapping algorithm incorporates a loop closure and map relaxation techniques to fix and massage path integration errors, thus creating more of a topological map than a metric map. A loop closure event only occurs when a threshold of consecutive local view cells matches the camera's input, thus allowing for a change in the pose data. So as to save original pose data, the relaxed map is saved to an "Experience Map" (see Fig. 7 for an illustration of the Experience Map Space), and the local view cells with accompanying pose cell data are stored in a connection matrix. Due to the topological nature of the Experience Map, transitions between experiences are stored, thus allowing for route planning to be possible.

The benefit that comes from this design is that it is a first step into implementing the functionality of some of the specialized, navigation and spatial awareness, brain cells in a mobile robot. The down side is that it has been shown that the competitive attractor network can be easily replaced by a filter system [19], which leads to substantial computational speedup. Additionally, even with pruning in the Experience Map, data storage and processing does appear to grow unbounded.

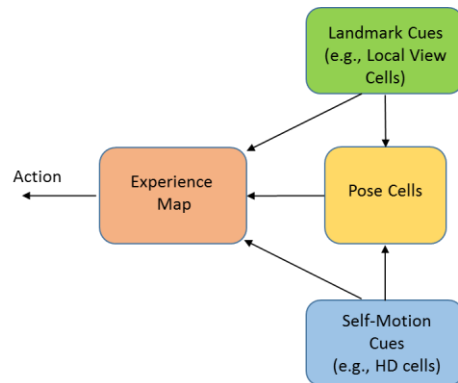


Fig. 6: Connectivity diagram of the RatSLAM, version 3.

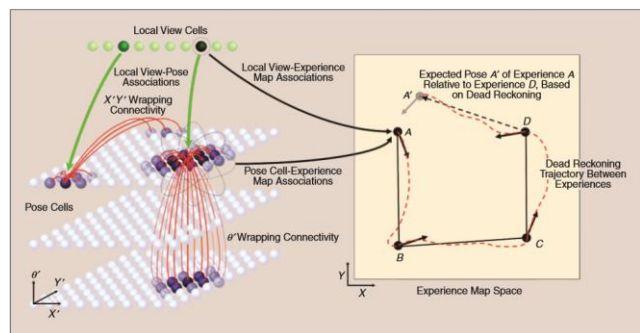


Fig. 7: The RatSLAM system. The left side represents the CAN system which forms pose cells from local view cells using a 3-D CAN algorithm. The right side represents the Experience Map, which helps disambiguate scenes that are similar in a semi-metric form. A further, detailed description can be found [20]. Permission to replicate given by Dr. Michael Milford.

## 4.2.2 Gaussier et al. '07

### 4.2.2.1 Transition Cell

Gaussier et al. built a neurobiologically inspired mobile robot navigation system in 2007 [55] using a new cell type they named the “transition cell”. Their cell is based on the concept of moving from one place cell to the next over a defined interval of time. Thus, two place cells are mapped to a single transition cell, creating a cell which represents both position and direction of movement or spatiotemporal transitions, thus a graph like structure.

### 4.2.2.2 Computation Complexity

Multiple neural networks span the system’s architecture, as shown in Fig. 8, from the landmark extraction/recognition stage to the cognitive map and motor transition stages. The many inputs of video, place cells, etc. into a system of neural networks, requires many calculations to be carried out during each time step. This complexity is similar to Arleo [38, 44], and Barrera and Weitzenfeld [2, 50, 54], covered in the previous section. To illuminate the amount of processing that is required it is stated in [55] that the system uses 3x dual core Pentium 4 Processors which run at 3 GHz each. Azimuth angles are measured using an on-board compass, displacement is obtained from wheel encoders, and the visual is obtained from a panoramic camera.

The navigation process starts at the left most part of Fig. 8, where a single, potential landmark is selected and analyzed at a given time. This occurs up to  $N$  times per snapshot, where  $N$  is a set to a value to help balance the algorithm’s efficiency with its robustness. Therefore, as expected in any visual extraction/recognition system, a fair amount of processing time and power is spent during this stage. Additionally, during the initial exploration phase, weighted neural network coefficients are calculated for each potential landmark (32 x 32 pixels) and azimuth grid value, so that these small local views can be learned online. For more detail on the calculations performed to arrive at the place cells from the landmark-azimuth matrix (PrPh) consult [55].

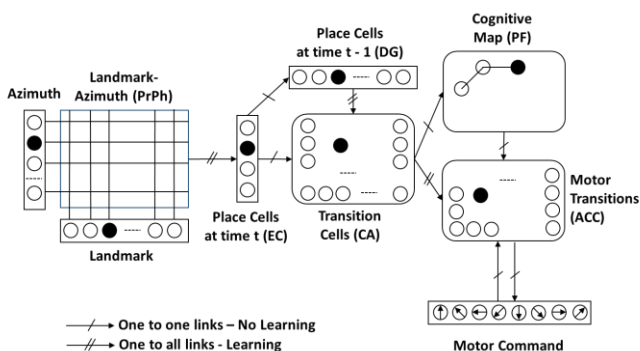


Fig. 8: The system’s neural network based model architecture. Processing flow starts at the far left with the input of each camera snapshot [55].

### 4.2.2.3 Cognitive Map

Each place cell (center of Fig. 8) is connected to each neuron of the landmark-azimuth matrix, where each connection has its own, unique, learned weights for that landmark-azimuth-place cell combination, as well as temporary scalars for the current, potential landmark view. However, it is very likely that several place cells will be active enough at a given location. The paper states that when a whole area has been mapped, during the initial exploration phase, the place cells are divided up into their own areas to eliminate these overlaps, see Fig. 9. Thus, creating a cognitive map.

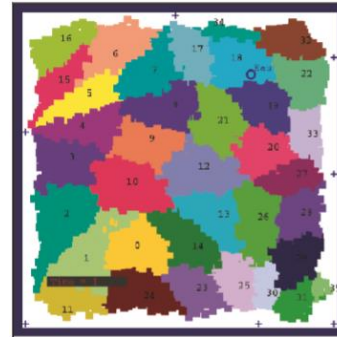


Fig. 9: Assignment of dedicated place cell fields. Permission to replicate given by Dr. Nicolas Cuperlier [55].

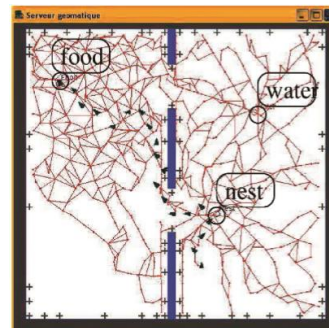


Fig. 10: Topographical cognitive map in the form of a graph is produced in the system, as illustrated. Permission to replicate given by Dr. Nicolas Cuperlier [55].

An assumption is made about the average number of possible place cell transitions from any particular place cell for the test conducted in [55]. This is done to reduce the  $N \times N$  neural network based, transition matrix to a  $6 \times N$ , where  $N$  represents the number of possible transition place cell targets. Thus, greatly reducing the computational complexity from  $O(N^2)$  to  $O(N)$ . However, this value may not work for all test cases, or in field use.

### 4.2.2.4 Route Planning

The robot’s cognitive map built during an initial exploration phase, as previously described, consists of nodes and edges, as show in Fig. 10, and is thus a graph:  $G = (N, E)$ . Each node is a transition cell and an edge signifies that the robot has traveled between the two transition cells or nodes. The edges hold weight value (e.g., function of use) and the nodes hold activity values. The recorded

nodes/edges of the cognitive map are used in a neural network version of the Bellman-Ford algorithm [56] to find the most direct route from a motivation point to the single source destination, while several types of motivations (drink, eat, sleep, etc.) are used to initiate the robot's travel to the proper destination source. The satisfaction level of the motivations changes with time and distance traveled, while increasing at the source.

### 4.3 Grid Cell Centric Systems

Perhaps due to the fact that the grid cell was not discovered until 2005, or due to its complex nature and unknown functionality/contribution to navigation, there is a sparse number of robot navigation systems that are based on the grid cell. Instead related research in grid cells come from computational/oscillational models [57-59].

As covered in the previous section, Milford and Wyeth [16, 17] use pose cells in their neurobiological based navigation model, which are similar conjunctive grid cells (as further described in [60]). Additionally, the wrapping connectivity of the pose cell grid creates a grid cell type pattern. However, the pose cell grid is laid out and used more like a discretized three dimensional graph rather than the known lay out of a grid cell's firing pattern. There certainly are similarities, but this work will remain in the theoretic cell section.

Additionally, Gaussier et al. [61, 62] used a grid cell mathematical model of the grid cell for their mobile robot navigation system. However, the grid cell is a modulo projection of the path integration input. The tests performed on the mobile robot shows poor patterns for the grid cell firing when relying on just path integration with growing accumulated errors as expected. Adding visual input to reset and recalibrate the path integration fixes the noisy path integration input, and thus sharpening the firing pattern of the grid cells. The grid cells are thus used more as a test pattern for various arenas and path integration degradation used. The grid cells do not add Cartesian mapping to the environment as might be expected, if it were used for mapping and route planning. Thus, this study does not fully fit this section and will not be covered in any more detail.

## 5 LITERATURE SURVEY ANALYSIS

As stated previously, the main focus of this paper is to present research on state of the art mobile robot navigations systems that are based on true rodent neurobiological spatial awareness and navigation brain cells. More specifically, this paper critiques how closely these navigation systems emulate neurobiological entities (e.g., posterior parietal cortex, dorsolateral medial entorhinal cortex, hippocampus, basal ganglia, place cells, head direction cells, etc.), the systems' autonomy classification, as well as their cognitive mapping and route planning capabilities. A summary of the answer to these questions can be found in Table I, as well as critiques at the end of each source surveyed. For completeness, a brief summary on the importance of visual cues to these systems, as well as a discussion on the Hebbian learning rule used in the literature and the computational

limitations of the scalability of these types of navigation systems due to the use of neural networks are covered.

### 5.1 Visual Cues

As discussed in Section 2 and exemplified in the literature summarized above, it is quite apparent that there is a strong correlation between the visual recognition capabilities and the overall navigation capabilities of the neurobiological based mobile robot. Navigation dominant on visual cues is referred to as taxon navigation, and applies to animals, humans, insects, etc., as well as classical and neurobiological based mobile robot navigation systems. This comes as no surprise as it has been shown that the specialized navigation and spatial awareness cells of a rodent are dependent to some degree on visual cues [20, 63-66]. Additionally, biological systems, such as those found in rodents, can navigate on non-visuals cues as well. These can be auditory, olfactory, and/or somatosensory cues. However, adding more allothetic sources to the neurobiological based navigation system and keeping to theme of using artificial neural networks (ANN), further taxes the on-board processing and memory systems, as discussed below.

### 5.2 Neural Networks

#### 5.2.1 Continuous Attractor Network

To keep on track with closely modelling a neurobiological system, both allothetic and idiothetic stimuli are fed into ANNs in all of the literature. The one difference is with the RatSLAM system [16, 17, 67, 68], which uses a variant of an ANN system called the (3-D) continuous attractor network (CAN) system (see Fig. 7). Although the CAN is a type of ANN, it is less computationally complex to update due to the fact that the activity values of the CAN units are varied between 0 and 1, thus keeping the weighted connections fixed. Changes in the CAN cell's activity level  $\Delta P$  is given in [17] by:

$$\Delta P = P * \varepsilon - \varphi, \quad (1)$$

or,

$$\Delta P_{x', y', \theta'} = \sum_i \sum_j \sum_k P_{i,j,k} \varepsilon_{a,b,c} - \varphi \quad (2)$$

where  $P$  represents the activity matrix of the network ( $P_{x', y', \theta'}$ ),  $\varepsilon$  is the connection matrix,  $*$  is the convolution operator, and the constant  $\varphi$  is used to create global inhibition and inhibition in the connection matrix. The matrices are fixed in size at the start of the robot's program.

Another difference between the RatSLAM system and the rest of the systems presented in the literature review section, is that the other systems use ANNs throughout their navigational system (thus increasing the computational complexity, but staying with the theme), while RatSLAM only uses the CAN for mobile robot pose determination. The visual snapshot matching appears to be of a non-ANN based algorithm.

### 5.2.2 Hebbian Learning Rule

Hebbian based ANN used in the research literature has the general equation of:

$$y_i = \sum_j w_{ij}x_j \quad (3)$$

and 
$$\Delta w_{ij} = \alpha x_j y_i \quad (4)$$

where,  $y_i$  is the output from neuron  $i$ ,  $x_j$  is the  $j^{\text{th}}$  input, and  $w_{ij}$  is the weight from  $x_j$  to  $y_i$ . The scalar  $\alpha$  is known as the learning rate and it may change with time. The Hebbian learning rule ( $\Delta w_{ij}$ ) is named after D. Hebb [46] and his theory that the connection or synapse between two neurons strengthen as a result of a repeated pre- and postsynaptic neuron firing relationship. Incorporating a bias or threshold term  $w_0$ , and some transfer function  $\sigma$  results in the Hebbian rule, as shown in [69-71], in the form of:

$$y_i = \sigma(\sum_j w_{ij}x_j - w_0) \quad (5)$$

The transfer function  $\sigma$  is typically a discrete step function:

$$\text{sgn}(t) = \begin{cases} 0 & \text{if } t < 0, \\ 1 & \text{if } t \geq 0, \end{cases} \quad (6)$$

or a smooth ‘‘sigmoid’’, e.g.

$$\sigma(t) = (1 + e^{-t})^{-1}, \quad (7)$$

The Hebbian general equation is inherently unstable, where all the synapses can either reach their maximum allowed value or transition to zero [72-74]. Thus, a simple alternative equation to (4), such as that used in [38], [39] and [75], is as follows:

$$\Delta w_{ij} = \alpha x_j y_i (1 - w_{ij}) \quad (8)$$

The neural networks used in the literature surveyed typically use no more than a single layer and are feedforward neural networks, see Fig. 11. These ANNs are adequate for simple, discrete input/output combinations, such as heading, turn angle, etc.

### 5.3 Computational Complexity Limiting Realism Scalability

When determining the computational complexity of a neural network, there are three important parameters to consider: *size*, *depth* and *weight* of the network. The *size* is the number of neurons, the *depth* is the length of a longest path from an input point to an output neuron, while the sum total of the absolute values of the weights represent the *weight* of the network.

The training of the ANNs that are used for complex pattern recognition, such as those found in interfacing allothetic stimuli to the navigation system, can really only be accomplished off-line. The processing power and time required would have too large of an impact on mobile robot

resources and usability. This is due to the many forward propagation and back propagation cycles required to set the weights of the ANN to the most optimum values possible (given set number of cycle constraints) for each training sample in the training phase. The time complexity will be a function of network *size* and particularly *depth*. An example of a simple two input, two output, single layer ANN is given in Fig. 11. Further examples can be found in the literature surveyed.

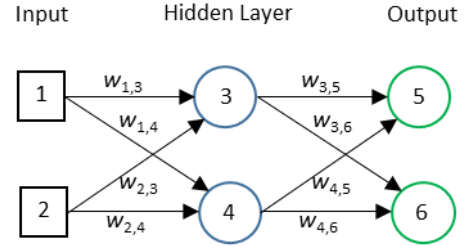


Fig. 11: Single layer ANN with two inputs, two outputs and two neurons.

Ways in which to add neurobiological based entities, such as allothetic stimuli, other percepts and/or controlling influences (e.g., nucleus accumbens, grid cells, etc) from various parts of the brain, while maintaining a usable mobile robot footprint, are as follows:

- 1) Use of mobile graphics processor units for massively parallel, general purpose computing (GPGPU) of more complex ANNs,
- 2) Removing ANNs from simpler parts of the system that can be easily replaced by a good, cheap sensor (e.g., head direction ANN in [38] with a MEMS gyroscope).
- 3) Creating an application specific integrated circuit (ASIC) that models ANNs.

Option 3 would be the most expensive, but also the most efficient in power, size and processing capabilities. Option 1 is a more flexible option, but still requires a great deal of power and special programming expertise. An example of what is available is the NVIDIA’s® Tegra® K1 Mobile GPU with 192 light weight parallel processor cores. They can be programmed using CUDA or cuDNN. Option 2 takes the system away from the realism of a neurobiological system, but some tradeoffs need to be made to model portions that are most important to the research.



**Table I: Neurobiological Based Navigation Research**

<i>Authors/Articles</i>	<i>Platform/Sensors</i>	<i>Visual Capabilities</i>	<i>Brain Cells Emulated</i>	<i>Cognitive Map</i>	<i>Route Planning and Autonomy (*)</i>
Arleo; Gerstner [38]	1) Khepera mobile robot platform. 2) 8 IR sensors – Obstacle detection. 3) Light detector – Ambient light measure. 4) Camera 90° H – Self-localization. 5) Odometer – Self-motion	- Offline, unsupervised, Hebbian learning, network (NN) training. - Four 90° horizontal snapshots taken (N, W, S, E) to create a single, location recognizable view. - Used primarily to assist with robot NN directionality.	- Place cells and - Head direction (HD) cells	Built into NNs of place cells & head direction cells. (Use of external homing light and offline NNs).	Not applicable.
Hafner [48]	1) Omnidirectional camera 2) Compass.	- 360° snapshot divided into 16 segments. Input into place cell NN, thus assists with robot's position determination.	-Place cells.	Topological map-Relational nav. connections between place cells.	Can only be performed in simulations due to amount of metric data processing required.
Barrera & Weitzenfeld [2, 50]	1) Sony AIBO, 4 legged robot. 2) Camera 50° H 3) Limited turns in increments of +/- 45°. 4) External PC w/ 1.8 GHz Pentium 4 Processor. Runs nav. model and connects. wirelessly to AIBO robot.	- Simple color recognition representing landmarks and goal. - Distance extracted from images of engineered environment and known relations.	-Places cells & many neuro-physiological based elements.	Place cells (nodes) and connections (edges). Simple T-maze and 8-arm maze.	Ability to learn and unlearn goal locations
Wyeth & Milford [16, 17]	1) Pioneer 2-DXE robot. 2) Motor encoders – Odometry 3) Sonar & laser range finder – Collision avoidance & pathway centering. 4) Panoramic camera syst. – Landmark recognition.	- 360° snapshot. Each unique snapshot is stored as a local view cell (VC) for landmark recognition	- Place cell & head direction cell combined as a pose cell.	A cognitive map is stored in an experience map. The map is created from the pose cells in the competitive attractor network (CAN).	Office delivery locations are stored in the mobile robot, which uses the exp. map and CAN to make deliveries. *Autonomous.
Gaussier et al. [55]	1) Robot with 3x Dual Core Pentium Processors (3 GHz each). 2) Panoramic camera. 3) Compass to measure azimuth angles. 4) Wheel encoders.	- 360° snapshot taken at low resolution and image is convolved using difference of Gaussian (DoG) to detect characteristic points (Landmark recognition).	- Place cells coupled together to create transition cells.	Topological map. Created online during initial exploration phase: images and directions used to create place cells which are then used to create trans. cells.	Use of the Bellman-Ford algorithm to choose most direct route from the cogn. map (transition cells with weighted links). *Autonomous.
Strösslin [39]	1) Khepera mobil robot platform. 2) Camera 60° H FOV. 3) Odometers. 4) Proximity sensors.	- Simulates rodent's FOV by rotating camera 4 times to obtain 240° FOV image. - Extracts directional information from visual inputs. - Path integration through visual and self-motion information.	- Place cells and - HD cells - Many neuro-physiological based elements.	Combined place code (CPC) neurons, where visual and odometric information is stored.	Results come partially from the robot and from simulation of agent. Thus, not applicable.

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