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# Analysis of Sensing Errors for Manufacturing Geometric Objects from Sensed Data *

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**Abstract.** In this work we address the problem of manufacturing machine parts from sensed data. Constructing geometric models for objects from sensed data is the intermediate step in a reverse engineering manufacturing system. Sensors are usually inaccurate, providing uncertain sensed information. We construct geometric entities with uncertainty models from noisy measurements for the objects under consideration, and proceed to do reasoning on the uncertain geometries, thus, adding robustness to the construction of geometries from sensed data.

**Key words:** manufacturing, tolerance, sensing, uncertainty modelling.

## 1. Introduction

Reverse engineering is a process that reconstructs a representation of a physical model, so that it can be reproduced identically. It is one of the new branches within the CAD/CAM field. Parts are manufactured according to blue prints, but when blue prints are not available, (for example, when the part is too old, and its blue prints are missing), reverse engineering can be used to reproduce these parts. This can be achieved by the following two major steps: sensing the part to construct its CAD representation and then manufacturing the part according to the representation. It is easy to see that the accuracy of the measurements is the key to succeed in eproducing an accurate CAD model.

The accuracy of the measurements can be improved not only by improving the quality of measuring instrument, but also by optimizing sampling data. A reverse engineering system has been built and the measuring process is done by a vision sensor (B/W CCD camera) and a coordinate measuring machine (CMM). The physical model is inspected by the cooperation between the observer camera and the probing CMM. The observer camera provides a high level (qualitative) description of the physical model, and the CMM complements the CAD model

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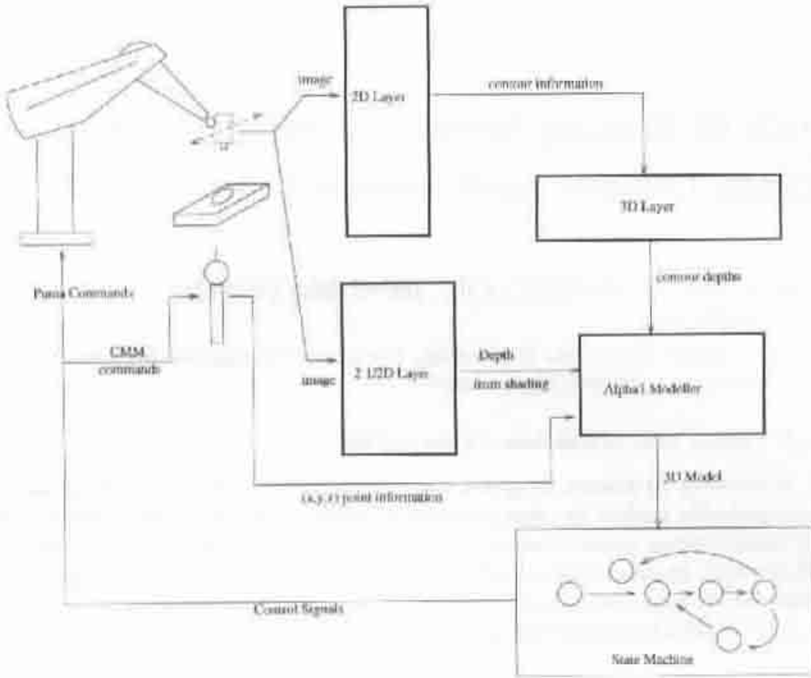


*Figure 1.* Overview of the system.

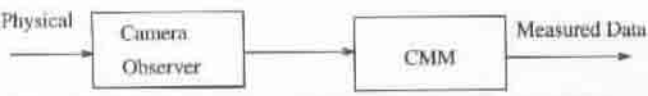


*Figure 2.* Sensing.

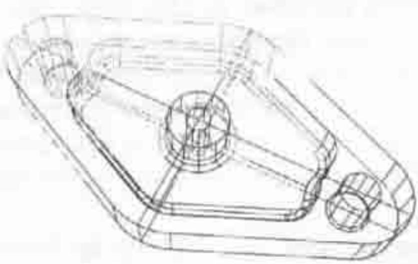

*Figure 3.* CAD model of the physical part.

with precise parametric data. Figures 1 and 2 provide an overview of the whole system. Figure 3 shows the CAD model of the physical parts. Figure 4 shows the vision setup.

In order to increase the accuracy and efficiency of the measurements, a feedback sensing system is designed as shown in Figure 5. In this feedback system, a


*Figure 4.* Experimental setup.

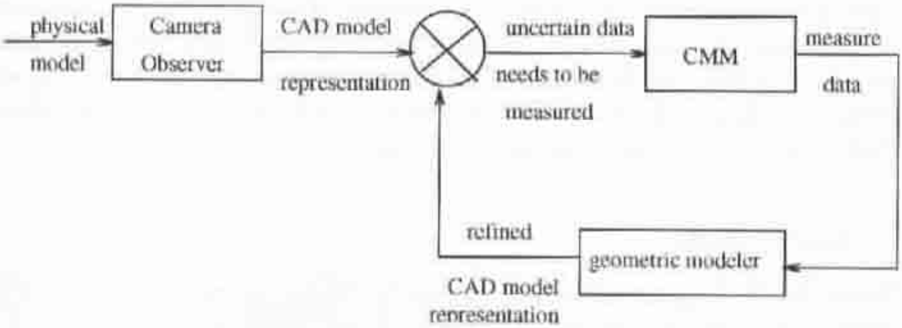


*Figure 5.* Feedback sensing system.

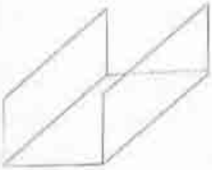

*Figure 6.* Slot.

probabilistic geometric modeler is utilized as a feedback agent to provide further measurements required to refine the CAD model, which also gives a quantitative measure of the accuracy of the current CAD model. The CMM machine actively measures the parameters for local features. By using the probabilistic geometric modeler performing modeling operations, redundant information of the part's geometry will be computed to reduce the load of the CMM measurement activities. Therefore, it improves the efficiency of the sensing process, besides, the geometric reasoning on the probabilities of uncertain geometries can guide the CMM to perform focused measurements to allow for higher accuracy and efficiency. For instance, the slot (see Figure 6) in mechanical engineering is a commonly used feature, and the parallelism of the two side planes is an important measurement.

The two side planes are based on sampling points from the CMM and/or visual data. Measurements of these points are not exact, therefore, these two planes constructed from measurements, are planes with the confidence measure as probabilities. Consequently, the parallelism is no longer a defined relation, it has a probability distribution. If the confidence of the parallelism does not satisfy the manufacturing requirements, refinement of the two side planes is required, hence, re-measuring of the points is performed.

Some work has been done in the probabilistic relationship between geometric objects and their relations, but the probability relations between the sampling points and geometric primitives have not yet been studied extensively. The geometric objects that this probabilistic geometric modeler is based on are constructed from sensed data. Therefore, the study of the relation of the probabilistic characteristics of geometric objects and the sensed data is very important. This paper presents the study of these relations. This work addresses the statistic geometric objects constructed from sensed data, relations of these statistic geometries, and the effect of decisions on the relative geometric objects.

## 2. Related Work

Stochastic geometry has been systematically studied by mathematicians. In [12], mathematical theories of stochastic geometry are well studied, and uncertain geometric features can be represented as constrained functions. Classic examples of stochastic geometry can be found in [11]. Kendall and Moran[12], describe a method of choosing distributions on geometric elements which provide a consistent interpretation of physical geometric elements.

Recently, research on sensing and uncertain geometry in robotics presents lots of ideas for handling uncertain geometry. Durrant-Whyte in [4, 5] has modeled the sensor in a manner that explicitly accounts for the inherent uncertainty encountered in robot operations. In Davidson's thesis [13], he made the important observation that arbitrary random geometric objects can be described by a point process in parameter space.

In computer-aided geometric modeling, methodologies for building a robust geometric modeler explores ways of handling the uncertain geometry caused by imprecise computations. Arbitrary decisions are made, when uncertainty arises. In [1, 2, 3, 6, 7, 8, 9, 10, 14], three region tolerances are used to keep track of uncertainty caused by the computational error. In [15], arbitrary decisions are made and corresponding uncertainties are restricted.

## 3. Representations for Uncertain Geometry

In geometric modeling, algorithms and representations for geometric objects are well developed, but the tolerance (uncertainty of geometry) has not yet been well defined. In [14], a geometric object is represented by boundary and hybrid repre-

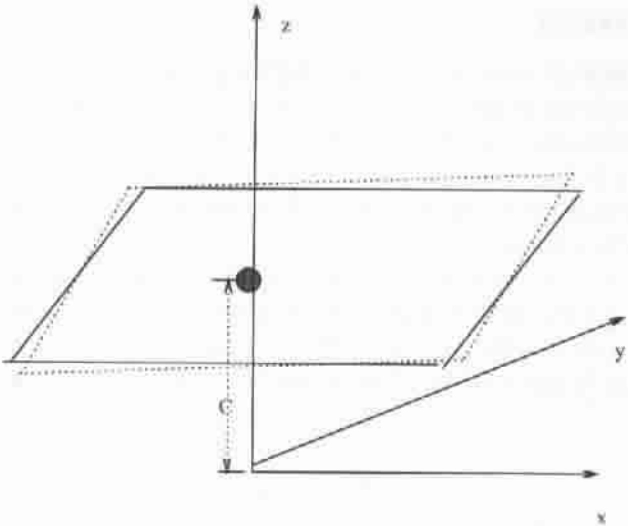


*Figure 7.* Representation of a plane.

sentations, associated with a tolerance representing the uncertainty of the geometry. We have developed a representation for uncertain geometry as follows.

An uncertain geometric object is represented in two parts: a geometric description, and a probabilistic distribution of geometry. The geometric description is a parameter vector, and the probabilistic distribution of geometry is a vector of the same dimension as the geometric description, but with the corresponding probabilistic distributions of the parameters.

For instance, a plane can be specified as an equation: $(A, B, C)$, $(f_a, f_b, f_c)$, where $(A, B, C)$ is the geometric description and $z = Ax + By + C$. $(f_a, f_b, f_c)$ is the probabilistic distribution of geometry, and also can be specified in another form: $(P, V)$, $(f_p, f_v)$, as shown in Figure 7, where $P$ is a base point, and $V$ is the normal vector of the plane. $f_p$ is the uncertainty of the base point, and $f_v$ is the uncertainty of the normal vector. It can be proved that $f_p$ and $f_v$ can be computed from $f_a$, $f_b$, $f_c$, and $P$, $V$ can be computed from $(A, B, C)$. By defining $f_a$, $f_b$, $f_c$, different types of probability distributions can be handled by this representation.

## 4. Experiments on Statistical Geometric Objects

The geometric objects that the modeler operates on, are constructed from the sensed data. The manner in which the distribution of sensed data affects the uncertainty of the geometry is the basis for defining the distributions of the geometry. In this section, the uncertainty of the plane related to the sensed 3D coordinates is studied. A set of discrete sensed data is used to perform the computations.

### 4.1. BEST LEAST SQUARE FIT

In order to reduce the random error, usually, $n$ sampling points are measured for defining a plane. Although the points have certain probability distributions which mainly depend on the measuring machine (e.g., CMM), they are independent random events. Therefore, a best least square fit method for computing the plane parameter is used. This approach yields the maximum likelihood result and confidence on the sampling data to be a plane.

The input data is $(x_i, y_i, z_i)$, where $x_i$, $y_i$, $z_i$ can have pre-chosen values or probability functions. They can be either independent, or correlated. Explicit function definition for a plane in 3D will be $z = Ax + By + C$. If there are $n$ points, the best least fit plane should be the solution of the following equation set:

$$Z = P \cdot X,$$

where

$$Z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}, \qquad P = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix}, \qquad X = \begin{bmatrix} A \\ B \\ C \end{bmatrix}.$$

Because $P$ is an $n \times 3$ matrix, and $X$ is a $3 \times 1$ matrix, $rank(P) = 3$, and $n \geqslant 3$, solution of $X$ is unique. When $n > 3$, the solution $X$ is a best least square fit $X = (P^{\mathrm{T}} \cdot P)^{-1} \cdot P \cdot Z$, or in the other form:

$$\begin{aligned} A &= f(x, y, z), \\ B &= g(x, y, z), \\ C &= h(x, y, z), \end{aligned}$$

where $x \in [x_1, x_2]$, $y \in [y_1, y_2]$, $z \in [z_1, z_2]$ are discrete. Exhaustively computing values of $f$, $g$, and $h$, will provide the discrete probability distribution arrays for $A$, $B$, and $C$.

From the above computations, we can see that the computation complexity is exponential. If $m$ is the number of distribution values and $n$ is the number of sampling points, this above computation will be performed $(3^m)^n$ times.

### 4.2. SENSED DATA AND THE CORRESPONDING RESULTS

The sensed data is modeled by discrete points with their corresponding probabilities. Normally, a point in 3D is represented as $(x, y, z)$, but for this sensing data, $x$, $y$, and $z$, are no longer a single value, they are distributions as shown in Figure 8.

Due to the computational complexity and the generality of the problem, three distribution values are randomly chosen for experiments. The resulting planes $(A, B, C)$ along with their distributions are computed. Graphs of $A$, $B$, and $C$ distributions are approximated by the following computations.

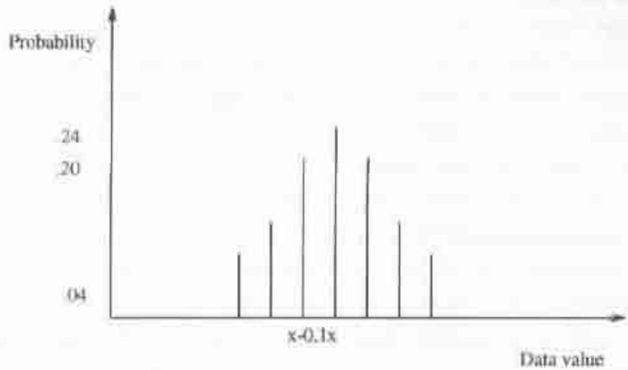


*Figure 8.* Sensed data.

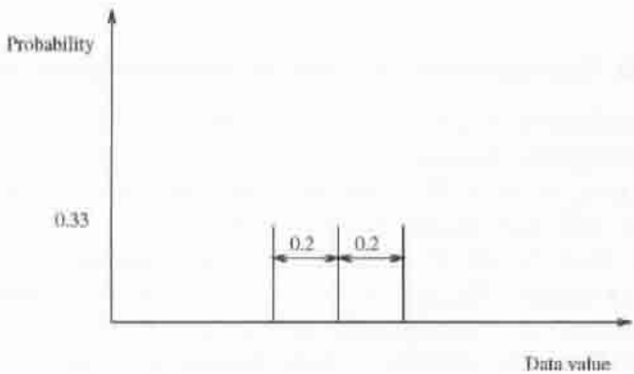


*Figure 9.* Uniform distribution.

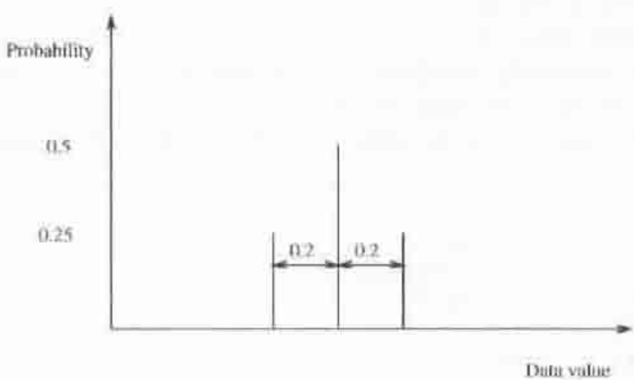


*Figure 10.* Gaussian distribution.

Capturing the essence of the $f(x)$ shape is very important. The computed data corresponds to the discrete state vector $(A, B, C)$ and its probability. $P(x_i < x < x_{i+1}) = \int_{x_i}^{x_{i+1}} f(x)\,dx$ is computed and plotted, where $x$ can be $A$, $B$, or $C$ and $x_{min} \leqslant x_i \leqslant x_{max}$. In order to smooth the curve, an overlapped set of $x_i$ is used. In the result figures, the $x$-axis are the values of $A$, $B$, $C$ respectively, and the $y$-axis are the corresponding probability of that value.

- Test 1. Uniform distribution: the sensing data is shown in Figure 9. There are a total of three points with such distributions, the planes defined by these points are computed. The distributions of $A$, $B$, $C$ are shown in the following figures.
- Test 2. Gaussian distribution: the sensing data is shown in Figure 10. There are a total of three points with such distributions, the planes defined by these points are computed. The distributions of $A$, $B$, $C$ are shown in figures 11–13. From the uniform and Gaussian distribution test data, it can be seen that the distribution of the $(a, b, c)$ space is Gaussian despite the probability distributions of the sensing data.

## 5. Relations of Statistic Geometries and its Effect on Relative Geometries

As mentioned in the introduction, the goal of this probabilistic modeler is to feedback control the sensing devices to measure the physical model and give a quantitative confidence measurement for the CAD model. Some relations of these uncertain geometries are computed with their uncertainty distributions.

Basically, geometric relations are set relations, such as: intersection, coincidence, incidence, and parallelism. Because of the uncertainty of the geometries, these relations are not defined, they are decisions with certain confidence that can be specified by its probability. For instance, a point incident to a plane, can be computed with 0.9 probability. This allows for reasoning based on probabilities.

A feedback computation of a plane that is supposed to be colinear with a given plane is studied. A program that takes the output discrete planes along with their probabilities is implemented, and the cases of parallel and colinear statements are computed with their probabilities. Some examples tested: the probability for parallelism is 0.824719, for colinearity it is 0.334722. The parallelism and colinearity of the planes of the three points Gaussian distribution and the uniform distributions have also been tested. The parallelism probability is 0.66730846, and the colinear-

*Table I.*

| $A$ | $B$ | $C$ | $P$ (probability) |
|---|---|---|---|
| 1.034723 | −0.961805 | 2.386458 | 0.33 |
| 1.036584 | −0.966461 | 2.391768 | 0.34 |
| 1.038042 | −0.970109 | 2.395926 | 0.33 |

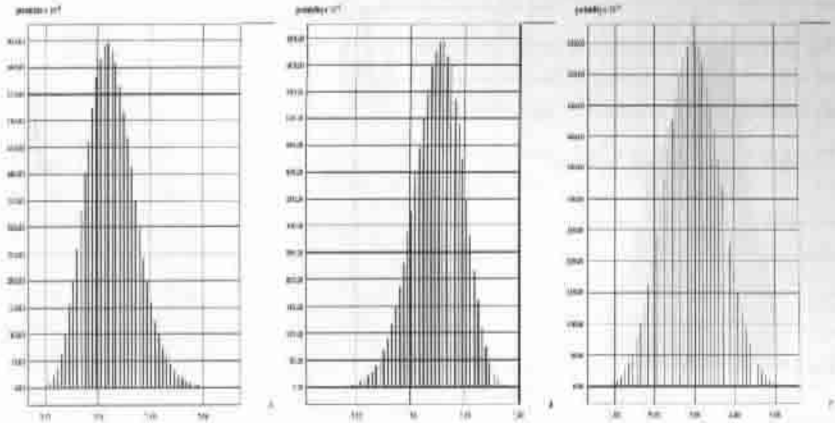

*Figure 11.* Parameter distribution of 3 points uniform sensing data.

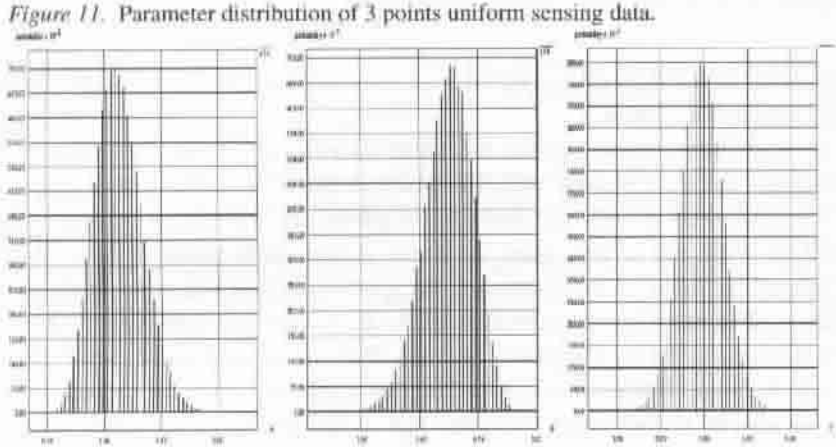

*Figure 12.* Parameter distribution of 3 points Gaussian sensing data.

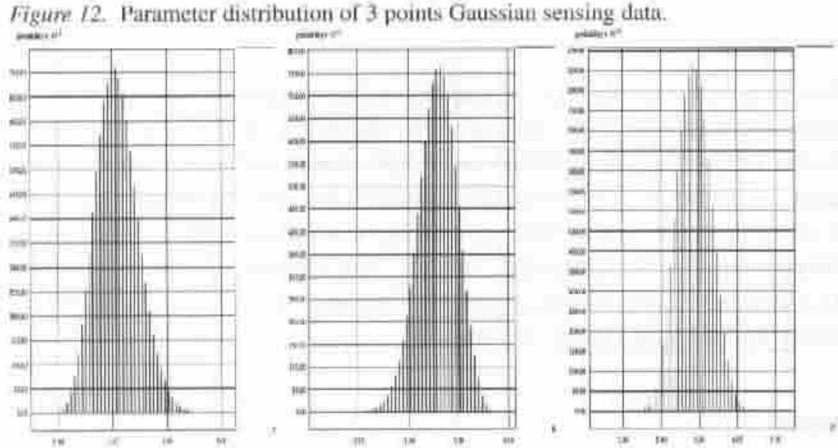

*Figure 13.* Parameter distribution of 4 points Gaussian sensing data.

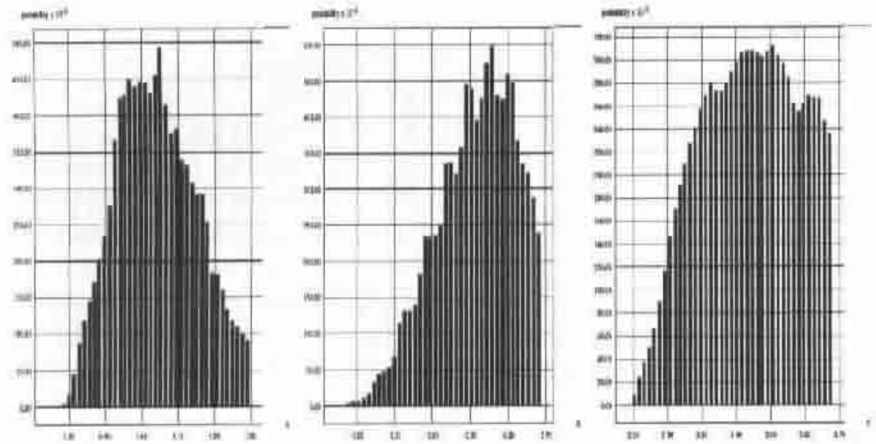

*Figure 14.* Redistribution of uniform sensing data.

ity probability is 0.27099140. (The tolerance for testing them is the square distance being less than $10e^{-2}$ mm.)

If the decision is made so that the plane constructed from the uniform distribution sensed data is colinear with the plane defined by Table I, then, its distribution is recomputed as follows: among this plane set, the plane instances that are not colinear with any of the plane instances in the given plane set are discarded. After discarding these plane instances, the distribution of the new plane set is renormalized. The resulting distributions of $A$, $B$, $C$ are shown in Figure 14. We can see that after recomputing the plane, the distributions of $A$, $B$ and $C$ are located in a more narrow range, further more, based on this redistributed plane set, the sampling points can also be recomputed and some can be discarded, or a re-measured.

## 6. Conclusions

Based on real sensing data, the probability of the geometry of objects under consideration is computed. This provides the capability of defining the probability distribution of the geometry based on robust computations as opposed to noisy measuring instruments. The relations between uncertain geometries are dependent on the uncertainty of geometries. Quantitative measurement for the constructed CAD model can thus be computed, and the relations can also involve the redistribution of the uncertainty of the geometry. This can be used as a feedback to guide the sensing and manufacturing modules.

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